Problem Based on Different Form of Equations (Factorization Method)

VERY SHORT ANSWER TYPE QUESTIONS :

VSA.1 Solve the equation : $(ax + b)^2 + (bx - a)^2 = 2(a^2 + b)^2$.

Sol. We have,

$$(ax + b)^2 + (bx - a)^2 = 2(a^2 + b^2).$$

$\Rightarrow$ $(a^2x^2 + b^2 + 2abx) + (b^2x^2 + a^2 - 2abx) = 2(a^2 + b^2).$

$\Rightarrow$ $(a^2 + b^2)x^2 = a^2 + b^2$

$\Rightarrow$ $x^2 = 1$

$\Rightarrow$ $x = \pm 1$ $\quad (\because a^2 + b^2 \neq 0)$

$\therefore$ The roots are -1 and 1.

VSA.2 Solve : $2x^2 + 1 = 0$.

Sol. We have,

$$2x^2 + 1 = 0$$

$\Rightarrow$ $2x^2 - (i)^2 = 0$ $\quad \Rightarrow \quad (\sqrt{2}x - i)(\sqrt{2}x + i) = 0$

Either $\sqrt{2}x - i = 0$ $\quad \Rightarrow \quad x = \frac{i}{\sqrt{2}} = \frac{\sqrt{2}}{2}i$

or $\sqrt{2}x + i = 0$ $\quad \Rightarrow \quad x = -\frac{i}{\sqrt{2}} = -\frac{\sqrt{2}}{2}i$

Hence, the roots of the given equation are $\frac{\sqrt{2}}{2}i$ and $-\frac{\sqrt{2}}{2}i$.

VSA.3 Solve the equation $x^2 - 4x + 13 = 0$ by factorization method.

Sol. We have,

$$x^2 - 4x + 13 = 0$$

$\Rightarrow$ $x^2 - 4x + 4 + 9 = 0$

$\Rightarrow$ $(x - 2)^2 + 9 = 0$

$\Rightarrow$ $(x - 2)^2 - 9i^2 = 0$

$\Rightarrow$ $(x - 2)^2 - (3i)^2 = 0$

$\Rightarrow$ $\{(x - 2) - 3i)\}\{(x - 2) + 3i)\} = 0$

$\Rightarrow$ $(x - 2 - 3i)(x - 2 + 3i) = 0$

$\Rightarrow$ $x - 2 - 3i = 0,$ or $x - 2 + 3i = 0$

$\Rightarrow$ $x = 2 + 3i,$ or $x = 2 - 3i$

Hence, the roots of the given equation are $2 + 3i$ and $2 - 3i$.

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VSA.4 Solve the following equation by factorization method :

$3x^2 + 7ix + 6 = 0$

Sol. We have,

$$3x^2 + 7ix + 6 = 0$$
$$\Rightarrow \quad 3x^2 + 9ix - 2ix - 6i^2 = 0$$
$$\Rightarrow \quad 3x(x + 3i) - 2i(x + 3i) = 0$$
$$\Rightarrow \quad (x + 3i)(3x - 2i) = 0$$
$$\Rightarrow \quad x + 3i = 0 \text{ or } 3x - 2i = 0$$
$$\Rightarrow \quad x = -3i \text{ or } x = \frac{2}{3}i$$

Hence, the roots of the given equation are $-3i$ and $\frac{2}{3}i$.

VSA.5 Solve the following equation by factorization method :

$x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$

Sol. We have,

$$x^2 - (3\sqrt{2} + 2i)x + 6\sqrt{2}i = 0$$
$$\Rightarrow \quad (x^2 - 3\sqrt{2}x) - (2ix - 6\sqrt{2}i) = 0$$
$$\Rightarrow \quad x(x - 3\sqrt{2}) - 2i(x - 3\sqrt{2}) = 0$$
$$\Rightarrow \quad (x - 2i)(x - 3\sqrt{2}) = 0$$
$$\Rightarrow \quad x - 2i = 0 \text{ or } x - 3\sqrt{2} = 0$$
$$\Rightarrow \quad x = 2i \text{ or } x = 3\sqrt{2}$$

Hence, the roots of the given equation are $2i$ and $3\sqrt{2}$.

VSA.6 Solve : $ix^2 - 4x - 4i = 0$.

Sol. We have,

$$ix^2 - 4x - 4i = 0$$
$$\Rightarrow \quad ix^2 - 2x - 2x - 4i = 0$$
$$\Rightarrow \quad ix(x + 2i) - 2(x + 2i) = 0$$
$$\Rightarrow \quad (x + 2i)(ix - 2) = 0$$

Either $x + 2i = 0$ or $ix - 2 = 0$

$$\Rightarrow \quad x = -2i \quad \text{or} \quad x = \frac{2}{i} = -2i$$

Hence, the roots are $-2i$ and $-2i$.

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VSA.7 Solve the equation by factorization method :

$2x^2 + 9ix + 5 = 0.$

Sol. We have,

$2x^2 + 9ix + 5 = 0$

$\Rightarrow \quad 2x^2 + 9ix - 5i^2 = 0 \qquad (\because i^2 = -1)$

$\Rightarrow \quad 2x^2 + 10ix - ix - 5i^2 = 0 \qquad [\because 2 \times 5 = 10 = 10 \times 1 \text{ and } 10 - 1 = 9]$

$\Rightarrow \quad 2x(x + 5i) - i(x + 5i) = 0$

$\Rightarrow \quad (x + 5i)(2x - i) = 0$

Either $x + 5i = 0$ or $2x - i = 0$

$\Rightarrow \quad x = -5i$ or $x = \frac{1}{2}i$

Hence, roots of the given equation are $-5i$ and $\frac{1}{2}i$.

VSA.8 Solve the equation : $ax^2 + bx + c = 0$, where $b = \ell + m$ and $\ell m = ac$.

Sol. We have,

$ax^2 + bx + c = 0$

$\Rightarrow \quad acx^2 + bcx + c^2 = 0$

$\Rightarrow \quad \ell mx^2 + (\ell + m)cx + c^2 = 0$

$\Rightarrow \quad (\ell mx^2 + \ell cx) + (mcx + c^2) = 0$

$\Rightarrow \quad \ell x(mx + c) + c(mx + c) = 0$

$\Rightarrow \quad (mx + c) + (\ell x + c) = 0$

$\therefore \quad mx + c = 0$ or $\ell x + c = 0$

$\Rightarrow \quad x = -\frac{c}{m}$ or $x = -\frac{c}{\ell}$

$\therefore$ The roots are $-\frac{c}{m}$ and $-\frac{c}{\ell}$.

VSA.9 Solve the equation : $x^2 + (3 - i)x - 3i = 0$.

Sol. We have,

$x^2 + (3 - i)x - 3i = 0$

$\Rightarrow \quad x^2 + 3x - ix - 3i = 0$

$\Rightarrow \quad x(x + 3) - i(x + 3) = 0$

$\Rightarrow \quad (x + 3)(x - i) = 0$

Either $x + 3 = 0$ or $x - i = 0$

$\Rightarrow \quad x = -3$ or $x = i$

Hence the roots are -3, i.

Problem Based on Different Form of Equations (Factorization Method)

SHORT ANSWER TYPE QUESTIONS :

SA.1 Solve the equation : $\frac{1}{p+q+x} = \frac{1}{p} + \frac{1}{q} + \frac{1}{x}$.

Sol. We have,

$$\frac{1}{p+q+x} = \frac{1}{p} + \frac{1}{q} + \frac{1}{x}$$

$$\Rightarrow \frac{1}{p+q+x} - \frac{1}{x} = \frac{1}{p} + \frac{1}{q}$$

$$\Rightarrow \frac{x-p-q-x}{(p+q+x)x} = \frac{q+p}{pq}$$

$$\Rightarrow \frac{-(p+q)}{(p+q+x)x} = \frac{p+q}{pq}$$

$$\Rightarrow \frac{-1}{(p+q+x)x} = \frac{1}{pq}$$

$\Rightarrow$ $-pq = (p + q + x)\, x$

$\Rightarrow$ $x^2 + px + qx + pq = 0$

$\Rightarrow$ $x(x + p) + q(x + p) = 0$

$\Rightarrow$ $(x + p)(x + q) = 0$

$\Rightarrow$ $x + p = 0$ or $x + q = 0$

$x + p = 0$ implies $x = -p$ and $x + q = 0$ implies $x = -q$.

$\therefore$ The roots are $-p$ and $-q$.

SA.2 Solve : $2y^2 + 6y + \frac{17}{2} = 0$

Sol. We have,

$$2y^2 + 6y + \frac{17}{2} = 0$$

$$\Rightarrow y^2 + 3y + \frac{17}{4} = 0$$

$$\Rightarrow (y)^2 + 2(y)\left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{17}{4} = 0$$

$$\Rightarrow \left(y + \frac{3}{2}\right)^2 + 2 = 0$$

$$\Rightarrow \left(y + \frac{3}{2}\right)^2 - (i\sqrt{2})^2 = 0$$

$\Rightarrow \quad \left(y + \frac{3}{2} + i\sqrt{2}\right)\left(y + \frac{3}{2} - i\sqrt{2}\right) = 0$

Either $\left(y + \frac{3}{2} + i\sqrt{2}\right) = 0 \quad \Rightarrow \quad y = -\frac{3}{2} - i\sqrt{2}$

$\Rightarrow \quad \left(y + \frac{3}{2} - i\sqrt{2}\right) = 0 \quad \Rightarrow \quad y = -\frac{3}{2} + i\sqrt{2}$

Hence, roots of the given equation are $-\frac{3}{2} - i\sqrt{2}$ and $-\frac{3}{2} + i\sqrt{2}$.

SA.3 Solve : $x^2 - 4x + 20 = 0$.

Sol. We have,

$x^2 - 4x + 20 = 0$

$\Rightarrow \quad (x)^2 - 2(x)(2) + (2)^2 - (2)^2 + 20 = 0$

$\Rightarrow \quad (x - 2)^2 + 16 = 0$

$\Rightarrow \quad (x - 2)^2 - (i4)^2 = 0$

$\Rightarrow \quad (x - 2 + i4)(x - 2 - i4) = 0$

Either $x - 2 + i4 = 0 \quad \Rightarrow \quad x = 2 - i4$

or $x - 2 - i4 = 0 \quad \Rightarrow \quad x = 2 + i4$

Hence, the roots of the given equation are $2 - i4$ and $2 + i4$.

SA.4 Solve : $\frac{p}{px-1} + \frac{q}{qx-1} = p + q$.

Sol. We have,

$$\frac{p}{px-1} + \frac{q}{qx-1} = p + q$$

This equation contains four terms, we try to form two groups of two terms each, such that one term is positive and the other is negative.

$\Rightarrow \quad \left(\frac{p}{px-1} - q\right) + \left(\frac{q}{qx-1} - p\right) = 0$

$\Rightarrow \quad \frac{p - pqx + q}{px-1} + \left(\frac{q - pqx + p}{qx-1}\right) = 0$

$\Rightarrow \quad (p - pqx + q)\left(\frac{1}{px-1} + \frac{1}{qx-1}\right) = 0$

Either $p - pqx + q = 0$ or $\frac{1}{px-1} + \frac{1}{qx-1} = 0$

$\Rightarrow \quad p + q = pqx \quad$ or $\quad \dfrac{(qx-1)+(px-1)}{(px-1)(qx-1)} = 0$

$\Rightarrow \quad x = \dfrac{p+q}{pq} \quad$ or $\quad qx - 1 + px - 1 = 0$

$\Rightarrow \quad x = \dfrac{p+q}{pq} \quad$ or $\quad (p + q)\,x = 2$ i.e. $x = \dfrac{2}{p+q}$

Therefore, the roots are $\dfrac{p+q}{pq}$ and $\dfrac{2}{p+q}$.

SA.5 Solve the equation : $\dfrac{x-p}{q} + \dfrac{x-q}{p} = \dfrac{q}{x-p} + \dfrac{p}{x-q}$

Sol. $\dfrac{x-p}{q} + \dfrac{x-q}{p} = \dfrac{q}{x-p} + \dfrac{p}{x-q}$

$\Rightarrow \quad \dfrac{x-p}{q} - \dfrac{p}{x-q} = \dfrac{q}{x-p} - \dfrac{x-q}{p}$

$\Rightarrow \quad \dfrac{(x-p)\,(x-q)-pq}{q\,(x-q)} = \dfrac{pq-(x-q)\,(x-p)}{(x-p)\,p}$

$\Rightarrow \quad (x - p)p\,[(x - p)\,(x - q) - pq] = [pq - (x - q)\,(x - p)]\,q(x - q)$

$\Rightarrow \quad (x - p)p\,[(x - p)\,(x - q) - pq] + (x - q)q\,[(x - p)\,(x - q) - pq] = 0$

$\Rightarrow \quad (x - p)\,(x - q) - pq\,[(x - p)p + (x - q)q] = 0$

Either $\quad (x - p)\,(x - q) - pq = 0 \quad$ or $\quad (x - p)p + (x - q)q = 0$

$\Rightarrow \quad x^2 - xq - xp + pq - pq = 0 \quad$ or $\quad xp - p^2 + xq - q^2 = 0$

$\Rightarrow \quad x^2 - (p + q)x = 0 \quad$ or $\quad (xp + xq) = p^2 + q^2$

$\Rightarrow \quad x(x - (p + q) = 0 \quad$ or $\quad x(p + q) = p^2 + q^2$

$\Rightarrow \quad x = 0 \quad$ or $\quad x = \dfrac{p^2+q^2}{p+q}$

$\Rightarrow \quad x = p + q \quad$ or $\quad x = \dfrac{p^2+q^2}{p+q}$