Problem Based on Different Form of Equations (Formula Method)

VERY SHORT ANSWER TYPE QUESTIONS :

VSA.1 Solve the equation : $x^2 - 4x + 7 = 0$

Sol. We have,

$x^2 - 4x + 7 = 0$

Here $a = 1, b = -4, c = 7$

Now, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore$ $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$

$= \dfrac{4 \pm 2\sqrt{3}\,i}{2} = 2 \pm \sqrt{3}\,i.$

VSA.2 Solve : $x^2 + 2|x| - 8 = 0.$

Sol. We have,

$x^2 + 2|x| - 8 = 0.$

$\therefore$ $|x|^2 + 2|x| - 8 = 0.$ $[\because |x|^2 = x^2]$

$\therefore$ $|x| = \dfrac{-2 \pm \sqrt{4 + 32}}{2}$

$= \dfrac{-2 \pm 6}{2} = -4, 2$

$|x| = -4$ is impossible and $|x| = 2$

$\Rightarrow$ $x = \pm 2.$

$\therefore$ Roots are $-2, 2$.

VSA.3 Solve the equation : $5x^2 - 4ix + 9 = 0.$

Sol. We have,

$5x^2 - 4ix + 9 = 0$

Here $a = 5, b = -4i, c = 9$

Now, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-4i) \pm \sqrt{(-4i)^2 - 4(5)(9)}}{2(5)}$

$= \dfrac{4i \pm \sqrt{-16 - 180}}{10}$

$= \dfrac{4i \pm \sqrt{-196}}{10} = \dfrac{4i \pm 14i}{10}$

$= \dfrac{18i}{10}, -\dfrac{10i}{10} = \dfrac{9}{5}i, -i.$

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VSA.4 Solve the following quadratic equations :

$25x^2 - 30x + 11 = 0.$

Sol. Comparing the coefficients of the equation $25x^2 - 30x + 11 = 0$ with the general form $ax^2 + bx + c = 0$, we get, $a = 26, b = -30, c = 11$.

$$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4 \times 25 \times 11}}{2 \times 25} = \frac{30 \pm \sqrt{900 - 1100}}{50}$$

$$= \frac{30 \pm 10\sqrt{2}\,i}{50} = \frac{3}{5} \pm \frac{\sqrt{2}}{5}i.$$

Hence, roots of the given equation are $\frac{3}{5} \pm \frac{\sqrt{2}}{5}i$.

VSA.5 Solve : $ix^2 + 4x - 5i = 0$

Sol. We have,

$ix^2 + 4x - 5i = 0$ ....(1)

Comparing (1) with general equation $ax^2 + bx + c = 0$, we get $a = i, b = 4, c = -5i$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(i)\,(-5i)}}{2(i)} = \frac{-4 \pm \sqrt{16 - 20}}{2i} = \frac{-4 \pm \sqrt{-4}}{2i}$$

$\Rightarrow \quad \alpha = \frac{-4 + 2i}{2i} = 2i + 1$ and $\beta = \frac{-4 - 2i}{2i} = 2i - 1$.

Hence, roots of the given equation are $2i + 1$ and $2i - 1$.

VSA.6 Solve the following quadratic equation : $2x^2 - 2\sqrt{3}x + \frac{21}{8} = 0$

Sol. Comparing the equation, $2x^2 - 2\sqrt{3}x + \frac{21}{8} = 0$ with the general form

$ax^2 + bx + c = 0$

We get, $a = 2$, $b = -2\sqrt{3}$ and $c = \frac{21}{8}$

$$x = \frac{-(-2\sqrt{3}) \pm \sqrt{12 - 4 \times 2 \times \frac{21}{8}}}{2 \times 2} = \frac{2\sqrt{3} \pm \sqrt{12 - 21}}{4}$$

$$x = \frac{2\sqrt{3} \pm \sqrt{-9}}{4}$$

$$\alpha = \frac{2\sqrt{3} + 3i}{4} = \frac{\sqrt{3}}{2} + \frac{3i}{4} \text{ and } \beta = \frac{2\sqrt{3} - 3i}{4} = \frac{\sqrt{3}}{2} - \frac{3i}{4}$$

Hence roots are $\frac{\sqrt{3}}{2} + \frac{3i}{4}$ and $\frac{\sqrt{3}}{2} - \frac{3i}{4}$.

Problem Based on Different Form of Equations (Formula Method)

LONG ANSWER TYPE QUESTIONS :

LA.1 Solve the equation : $2x^2 - (3 + 7i)x + (9i - 3) = 0$.

Sol. We have,

$$2x^2 - (3 + 7i)x + (9i - 3) = 0$$

Here $a = 2,\ b = -(3 + 7i),\ c = 9i - 3$.

Now, $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 + 7i \pm \sqrt{[-(3+7i)]^2 - 4(2)\,(9i - 3)}}{2 \times 2}$$

$$x = \frac{3 + 7i \pm \sqrt{9 + 49i^2 + 42i - 72i + 24}}{4}$$

$\therefore$ $$x = \frac{3 + 7i \pm \sqrt{-16 - 30i}}{4} \qquad ....(1)$$

Let square roots of

$-16 - 30i = a + ib$.

$\therefore$ $-16 - 30i = (a + ib)^2$.

$\therefore$ $-16 - 30i = (a^2 - b^2) + (2ab)i$

$\Rightarrow$ $a^2 - b^2 = -16$ and $2ab = -30$.

Now, $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$.

$\therefore$ $(a^2 + b^2)^2 = (-16)^2 + (-30)^2 = 1156$.

$\therefore$ $a^2 + b^2 = 34$. $[\because a^2 + b^2 \geq 0]$

Also, $a^2 - b^2 = -16$.

Adding, we get

$2a^2 = 18,\ a^2 = 9,\ a = \pm 3$.

$a = 3 \quad \Rightarrow \quad b = \dfrac{-30}{2(3)} = -5$

$\therefore$ $a + ib = 3 - 5i$

$a = -3 \quad \Rightarrow \quad b = \dfrac{-30}{2(-3)} = 5$

$\therefore$ $a + ib = -(3 - 5i)$

$\therefore$ $a + ib = \pm(3 - 5i)$

$\therefore$ Square roots of $-16 - 30i = \pm(3 - 5i)$

$\therefore$ (1) implies

$$x = \frac{3 + 7i \pm (3 - 5i)}{4}$$

$$= \frac{3 + 7i + 3 - 5i}{4},\ \frac{3 + 7i - 3 + 5i}{4}$$

$$= \frac{3 + i}{2},\ 3i.$$