NON-UNIFORM CIRCULAR MOTION

If speed of particle during circular motion changes with time, then motion is termed as non-uniform circular motion.

Here, acceleration is there due to two types of changes in velocity vector :

(i) Acceleration due to change in only direction component of $\vec{v}$ $\Rightarrow$ centripetal acceleration

(ii) Acceleration due to change in only magnitude component of $\vec{v}$ $\Rightarrow$ tangential acceleration.

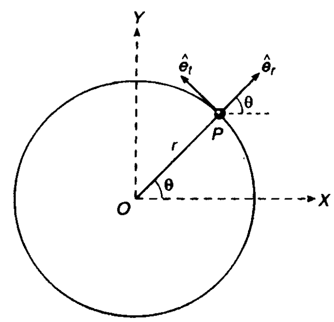
$\therefore$ Net acceleration in circular motion = $\vec{a}_{net} = \vec{a}_c + \vec{a}_t \Rightarrow |a_{net}| = \sqrt{|a_c|^2+|a_t|^2}$.

Here, both magnitude and direction of velocity change.

UNIT VECTORS ALONG THE RADIUS AND THE TANGENT

Consider a particle P moving in a circle of radius r and centred at origin O. The angular position of the particle at some instant is say θ. Let us define two unit vectors, one is $\hat{e}_r$ (called radial unit vector) which is along OP and the other is $\hat{e}_t$ (called the tangential unit vector) which is perpendicular to OP. Now, since

$$|\hat{e}_r| = |\hat{e}_t| = 1$$



We can write these two vectors as

$$\hat{e}_r = \cos\theta\ \hat{i} + \sin\theta\ \hat{j}$$

and $$\hat{e}_t = -\sin\theta\ \hat{i} + \cos\theta\ \hat{j}$$

VELOCITY AND ACCELERATION OF PARTICLE IN CIRCULAR MOTION

The position vector of particle P at the instant shown in figure can be written as

$$\vec{r} = \vec{OP} = r\hat{e}_r$$

or $$\vec{r} = r(\cos\theta\ \hat{i} + \sin\theta\ \hat{j})$$

The velocity of the particle can be obtained by differentiating $\vec{r}$ with respect to time t. Thus,

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}[r(\cos\theta\ \hat{i} + \sin\theta\ \hat{j})]$$

$$= r\left[\left(-\sin\theta\frac{d\theta}{dt}\right)i + \left(\cos\theta\frac{d\theta}{dt}\right)j\right]$$

or $$\vec{v} = r\omega\,[-\sin\theta\ \hat{i} + \cos\theta\ \hat{j}] \quad ....(i) \quad \left(\because \frac{d\theta}{dt} = \omega\right)$$

$\therefore \quad \vec{v} = r\omega\,\hat{e}_t$

Thus, we see that velocity of the particle is r ω along $\hat{e}_t$ or in tangential direction. Acceleration of the particle can be obtained by differentiating eqn. (i) with respect to time t. Thus,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left[r\omega\,(-\sin\theta\;\hat{i} + \cos\theta\;\hat{j}\right]$$

$$= r\left[\omega\frac{d}{dt}(-\sin\theta\hat{i} + \cos\theta\hat{j}) + \frac{d\omega}{dt}(-\sin\theta\hat{i} + \cos\theta\hat{j})\right]$$

$$= \omega r\left[-\cos\theta\frac{d\theta}{dt}\hat{i} - \sin\theta\frac{d\theta}{dt}j\right] + r\,\frac{d\omega}{dt}\,\hat{e}_t$$

$$\vec{a} = -\,\omega^2 r\hat{e}_r + \frac{dv}{dt}\,\hat{e}_t$$

so, $\quad a_c = -\,\omega^2 r\hat{e}_r \;,\; |a_c| = \omega^2 r = \frac{v^2}{r}$

and $\quad a_t = \frac{dv}{dt}\;\hat{e}_t\;,\; |a_t| = \frac{dv}{dt}$ (rate of change of speed)

Thus, acceleration of a particle moving in a circle has two components :

(a) $\hat{e}_t$ (along tangent) [called the tangential acceleration (a_t)]

(b) - $\hat{e}_r$ (or towards centre) [called radial or centripetal acceleration (a_r)]

Here, the two components are mutually perpendicular. Therefore, net acceleration of the particle will be :

$$a = \sqrt{a_c^2 + a_t^2}$$

$$= \sqrt{(r\omega^2)^2 + \left(\frac{dv}{dt}\right)^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

Ex.1 *The speed of a particle moving in a circle of radius r = 2m varies with time t as v = t^2, where t is in seconds & v is in m/sec. The radial acceleration at t = 2 sec is :*

(A) 2 m/s² *(B) 4 m/s²* *(C) 6 m/s²* *(D) 8 m/s²*

Sol. (D)

Linear speed of the particle at t = 2s is

$v = (2)^2 = 4$ m/sec

$\therefore$ Radial acceleration, $a_r = \frac{v^2}{r} = \frac{(4)^2}{2} = 8\ m/s^2$

Ex.2 *In the previous question, the tangential acceleration is -*

(A) 2 m/s² *(B) 4 m/s²* *(C) 6 m/s²* *(D) 8 m/s²*

Sol. (B)

Tangential acceleration is

$$a_t = \frac{dv}{dt} = 2t$$

$\therefore$ At t = 2 sec, $a_t = 2 \times 2 = 4$ m/s²

Ex.3 *In the previous question, the net acceleration is :*

(A) $2\sqrt{5}$ m/s² *(B) $4\sqrt{5}$ m/s²* *(C) $6\sqrt{5}$ m/s²* *(D) $8\sqrt{5}$ m/s²*

Sol. (B)

Net acceleration of particle at t = 2 sec is

$$a = \sqrt{(a_r)^2 + (a_t)^2} = \sqrt{(8)^2 + (4)^2}$$

$$= \sqrt{80} = 4\sqrt{5} \text{ m/sec}^2.$$

IMPORTANT POINTS

1. In uniform circular motion, speed (v) of the particle is constant i.e. $\frac{dv}{dt} = 0$. Thus,

$a_t = 0$ and $a_c = a_r = r\omega^2$

2. In accelerated circular motion, $\frac{dv}{dt}$ = positive i.e. a_t is along $\hat{e}_t$ or tangential acceleration of particle is parallel to velocity $\vec{v}$ because $\vec{v} = r\,\omega\hat{e}_t$ and $\vec{a}_t = \frac{dv}{dt}\hat{e}_t$

3. In decelerated circular motion, $\frac{dv}{dt}$ = negative and hence, tangential acceleration is antiparallel to velocity $\vec{v}$.

	UNIFORM CIRCULAR MOTION	NON UNIFORM CIRCULAR MOTION
(a)	Particle moves in circle with constant speed.	Speed also changes
(b)	$\vec{v}$ is changing (due to direction only) $\therefore \frac{d\|v\|}{dt} = 0 \qquad \therefore a_t = 0$	$a_t = \frac{d\|v\|}{dt} = \alpha \; r \; \hat{e}_t$ $a_r = - \; \omega^2 \; r \; \hat{e}_r$
(c)	$\vec{a}_{total} = \vec{a}_r = - \; \omega^2 \; r \; \hat{e}_r$ towards centre	$\|\vec{a}_{total}\| = \sqrt{\|\vec{a}_r\|^2 + \|\vec{a}_t\|^2} = \sqrt{\left\|\frac{v^2}{r}\right\|^2 + \left\|\frac{dv}{dt}\right\|^2}$
(d)	$\tan \alpha = 0$ as $\alpha = 0$	$\tan \alpha = \frac{a_t}{a_r} = \frac{dv/dt}{v^2/r}$

(where, α = angle between tangential acceleration and radial acceleration)

Ex.1 *A particle moves in a circle of radius 20 cm. It's linear speed is given by v = 2t, where t is in sec & v is in m/s. Find the radial and tangential accelerations at t = 3s. Also find net acc. at that time.*

Sol. $v_{t=3} = 2t = 6$ m/s $\qquad \therefore a_r = \frac{v^2}{r} = \frac{6x6}{0.2} = 180$ m/s^2 towards centre.

$a_t = \frac{dv}{dt} = \frac{d}{dt}(2t) = 2$ m/s^2 all the time

so $\quad a_{net} = -180 \; \hat{e}_r + 2 \; \hat{e}_t$, $|a_{net}| = \sqrt{180^2 + 2^2} = \sqrt{32404}$ m/s^2

Ex.2 *A particle is moving in a circular path with const. speed v. Find Δv from A to B if $\angle$ AOB = 40º. (Where A & B are points on circumference and O is the centre of circle)*

Sol. $\Delta v = \vec{v}_2 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1) = 2 \; v \cos (90º - 20º) = 2v \sin 20º$

or $\quad \Delta v = \sqrt{v^2 + v^2 - 2v^2 \cos(180º - 40º)}$

$= \sqrt{2v^2(1 - \cos 140º)} = \sqrt{2v^2 . 2 \sin^2 70º}$

$= 2 \; v \sin 70º$ Ans.

RELATIVE MOTION IN CIRCULAR MOTION

If two particles are moving on same circle or different coplanar concentric circles in same direction with different uniform angular speeds ω_A and ω_B respectively, the angular velocity of B relative to A for an observer at the centre will be

$$\omega_{rel} = \omega_B - \omega_A$$

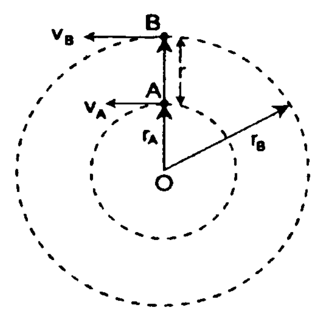
So, the time taken by one to complete one revolution around O with respect to the other (i.e. time, in which B completes one more or less revolution around O than A).

$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1 T_2}{T_1 - T_2} \qquad \left[\text{as } T = \frac{2\pi}{\omega}\right]$$

If two particles are moving on two different concentric circles with different velocities, then angular velocity of B relative to A as observed by A will depend on their positions and velocities. The general theory is beyond the scope of this book. However, to make this concept clear, consider the case when A and B are closest to each other and moving in same direction. In this situation,

$$v_{rel} = |\vec{v}_B - \vec{v}_A| = v_B - v_A$$

$$r_{rel} = |\vec{r}_B - \vec{r}_A| = r_B - r_A$$



So $$\omega_{rel} = \frac{v_{rel}}{r_{rel}} = \frac{v_B - v_A}{r_B - r_A}$$

NOTE : Here, only kinematics portion of circular motion is covered. Complete circular motion topics like motion in vertical circle, friction and banking of circular track, conical pendulum etc. will be discussed before the chapter Rotational Dynamics.

MISCELLANEOUS EXAMPLES BASED ON NON-UNIFORM CIRCULAR MOTION

Ex.1 A wheel is subjected to uniform angular acceleration about its axis. Initially, its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1. In the next 2 sec, it rotates through an additional angle θ_2. The ratio of θ_2/θ_1 is-

(A) 1 (B) 2 (C) 3 (D) 5

Sol. (C)

Using relation :

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta_1 = \frac{1}{2}(\alpha)(2)^2 = 2\alpha \qquad ....(1) \qquad [\text{As } \omega_0 = 0,\ t = 2 \text{ sec}]$$

Now using same equation for t = 4 sec, $\omega_0 = 0$

$$\theta_1 + \theta_2 = \frac{1}{2}(\alpha)(4)^2 = 8\alpha \qquad ....(2)$$

From (1) and (2),

$\theta_1 = 2\alpha$ and $\theta_2 = 6\alpha$

$$\therefore \quad \frac{\theta_2}{\theta_1} = 3.$$

Ex.2 If the equation for the displacement of a particle moving on a circular path is given by $\theta = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2 sec from its start is-

(A) 8 rad/sec (B) 12 rad/sec (C) 24 rad/sec (D) 36 rad/sec

Sol. (C)

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}(2t^3 + 0.5) = 6t^2$$

PRACTICE QUESTIONS BASED ON NON-UNIFORM CIRCULAR MOTION

Q.1 A fan is making 600 revolutions per minute. If after some time, it makes 1200 revolutions per minute, then increase in its angular velocity is-

(A) 10π rad/sec (B) 20π rad/sec (C) 40π rad/sec (D) 60π rad/sec

Q.2 For a particle in a non-uniformly accelerated circular motion-

(A) Velocity is radial and acceleration is transverse only.
(B) Velocity is transverse and acceleration is radial only.
(C) Velocity is radial and acceleration has both radial and transverse components.
(D) Velocity is transverse and acceleration has both radial and transverse components.

ANSWERS : **1.** (B) **2.** (D)

LEVEL # 1

Q.1 A body revolving in a circle with uniform speed possesses :
(A) Normal acceleration (B) Uniform acceleration
(C) Tangential acceleration (D) None of these

Q.2 A stone of mass m is tied to a string of length ℓ and rotated in a circle with a constant speed v, if the string is released, the stone flies :
(A) radially outward (B) radially inward
(C) tangentially outward (D) with an acceleration $\frac{mv^2}{\ell}$

Q.3 If a particle moves in a circle, describing equal angles in equal times, its velocity vector :
(A) remains constant (B) changes in magnitude
(C) changes in direction (D) changes both in magnitude and direction

Q.4 A particle of mass m moves in a circle of radius r with a uniform speed v; then its velocity vector is along :
(A) the radius (B) the axis of rotation
(C) the tangent (D) some other direction

Q.5 In the Q.4, the angular velocity is along :
(A) the radius (B) the axis of rotation
(C) the tangent (D) some other direction

Q.6 In the Q.4, the centripetal acceleration is along :
(A) the radius towards the centre (B) the radius away from the centre
(C) the tangent (D) the axis of rotation

Q.7 In the Q.4, the angular acceleration is :
(A) $\frac{v}{r}$ (B) $\frac{v^2}{r}$ (C) Zero (D) $\frac{r}{v}$

Q.8 In the Q.4, the angular velocity is :
(A) $\frac{v}{r}$ (B) vr (C) $\frac{v^2}{r}$ (D) $\frac{r}{v}$

Q.9 In the Q.4, the tangential acceleration is :
(A) $\frac{v}{r}$ (B) Zero (C) $\frac{v^2}{r}$ (D) $\frac{r}{v}$

Q.10 When a mass is rotating in a plane about a fixed point, its angular momentum is directed along:
(A) The radius
(B) The tangent to the orbit
(C) A line perpendicular to the plane of rotation
(D) None of the above

Q.11 The shaft of a motor rotates at constant angular velocity of 3000 rpm. The radians it has turned in 1 sec are :

(A) 1000π (B) 100π (C) π (D) 10π

Q.12 The angular speed of the second's hand of a clock is :

(A) $\frac{1}{6}$ rad/s (B) $\frac{\pi}{60}$ rad/s (C) $\frac{2\pi}{60}$ rad/s (D) $\frac{360}{60}$ rad/s

Q.13 A motor car is travelling at 60 m/s on a circular road of radius 1200 m. It is increasing its speed at the rate of 4 m/s^2. The acceleration of the car is :

(A) 3 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 7 m/s^2

Q.14 Two particles P and Q are located at distances r_P and r_Q respectively from the centre of rotating disc such that $r_P > r_Q$:

(A) both P and Q have the same acceleration
(B) both P and Q do not have any acceleration
(C) P has greater acceleration than Q
(D) Q has greater acceleration than P.

Q.15 Which of the following statements about the rotatory motion along a circular path is true ?

(A) Linear velocity is uniform when angular velocity is uniform
(B) Magnitude of acceleration is constant
(C) Acceleration is directed along the tangent to the circular path
(D) None of the above statements is correct

Q.16 The angular acceleration of a particle moving along a circular path with uniform speed is :

(A) uniform but non-zero
(B) zero
(C) variable
(D) such as cannot be predicted from the given information

Q.17 A particle covers equal distance around a circular path, in equal intervals of time. Which of the following quantities connected with the motion of the particle remains constant with time?

(A) Displacement (B) Velocity (C) Speed (D) Acceleration

Q.18 A particle covers equal distances around a circular path in equal intervals of time. It has uniform non-zero rate of change of :

(A) linear displacement (B) angular displacement
(C) linear velocity (D) angular velocity

Q.19 A particle is moving along a circular path. The angular velocity, linear velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are $\vec{\omega}$, $\vec{v}$, $\vec{\alpha}$ and $\vec{a}_c$. Which of the following relations is not correct ?

(A) $\vec{\omega} \perp \vec{v}$ (B) $\vec{\omega} \perp \vec{\alpha}$ (C) $\vec{\omega} \perp \vec{a}_c$ (D) $\vec{v} \perp \vec{a}_c$

Q.20 A particle is moving along a circular path with uniform speed. What is the angle between its instantaneous velocity and acceleration ?
(A) 0º (B) 45º (C) 90º (D) 180º

Q.21 A particle is moving along a circular path of radius 2m with uniform speed of 5 ms^{-1}. What will be the change in velocity when the particle completes half of the revolution ?
(A) Zero (B) 10 ms^{-1} (C) $10\sqrt{2}$ ms^{-1} (D) $\frac{10}{\sqrt{2}}$ ms^{-1}

Q.22 A car moving on a horizontal road may be thrown out of the road in taking a turn :
(A) by the gravitational force
(B) due to the lack of proper centripetal force
(C) due to the rolling frictional force between the tyre and road
(D) due to the reaction of the ground.

Q.23 A particle revolves with uniform velocity round a circular path. The acceleration of the particle is :
(A) along the circumference of the circle (B) along the tangent
(C) along the radius (D) zero

Q.24 A wheel is subjected to uniform angular acceleration about its axis. Initially it's angular velocity is zero. In the first 4 sec, it rotates through an angle θ_1. In the next 2 sec, it rotates through an additional angle θ_2. The ratio θ_2/θ_1 is :
(A) 1/4 (B) 3/4 (C) 5/4 (D) 7/4

Q.25 A motor car is travelling at 30 m/sec on a circular road of radius 500 m. It is increasing its speed at the rate of 2.0 ms^{-2}. The total acceleration is :
(A) 1.8 ms^{-2} (B) 2 ms^{-2} (C) 3.8 ms^{-2} (D) 2.7 ms^{-2}

Q.26 An electric fan has blades of length 30 cm as measured from the axis of rotation. If the fan is rotating at 1200 rpm, the acceleration of a point on the tip of the blade is about :
(A) 1600 ms^{-2} (B) 4740 ms^{-2} (C) 2370 ms^{-2} (D) 5055 ms^{-2}

Q.27 The acceleration of a train travelling with speed of 400 m/s as it goes round a curve of radius 160 m, is-
(A) 1 km/s^2 (B) 100 m/s^2 (C) 10 m/s^2 (D) 1 m/s^2

Q.28 The normal component of acceleration of a particle in circular motion is due to :
(A) speed of the particle (B) change in direction of velocity
(C) change in the magnitude of velocity (D) rate of change of acceleration

Q.29 The tangential component of acceleration of a particle in circular motion is due to :
(A) speed of the particle (B) change in the direction of velocity
(C) change in the magnitude of velocity (D) rate of change of acceleration

CIRCULAR MOTION

Q.30 A car of mass m moves in a horizontal circular path of radius r metre. At an instant its speed is V m/s and is increasing at a rate of a m/sec^2. Then the acceleration of the car is :

(A) $\frac{V^2}{r}$ (B) a (C) $\sqrt{a^2+\left(\frac{V^2}{r}\right)^2}$ (D) $\sqrt{a+\frac{V^2}{r}}$

Q.31 A body of mass 4 kg is being rotated with 120 rev per minute in a horizontal circular path of radius 2m. Its kinetic energy is :

(A) 2 J (B) 32 J (C) 80 J (D) 1263 J

Q.32 The average acceleration vector (taken over a full circle) for a particle having a uniform circular motion is :

(A) a constant vector of magnitude $\frac{V^2}{r}$

(B) a null vector

(C) a vector of magnitude $\frac{V^2}{r}$ directed normal to the plane of the given uniform circular motion

(D) equal to the instantaneous acceleration vector at the start of the motion.

Q.33 A body is moving in a circular path with acceleration a. If it's velocity gets doubled, find the ratio of acceleration after and before the change :

(A) 1 : 4 (B) $\frac{1}{2}$: 1 (C) 2 : 1 (D) 4 : 1

Q.34 When a body moves with a constant speed along a circle-

(A) No work is done on it (B) No acceleration is produced in the body

(C) No force acts on the body (D) Its velocity remains constant

Q.35 A wheel completes 2000 revolutions to cover the 9.5 km. distance, then the diameter of the wheel is-

(A) 1.5 m (B) 1.5 cm (C) 7.5 cm (D) 7.5 m

Q.36 A cycle wheel of radius 0.4 m completes one revolution in one second, then the acceleration of a point on the cycle wheel will be-

(A) 0.8 m/s^2 (B) 0.4 m/s^2 (C) 1.6π^2 m/s^2 (D) 0.4π^2 m/s^2

Q.37 What is the value of linear velocity, if $\vec{\omega} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{r} = 5\hat{i} - 6\hat{j} + 6\hat{k}$

(A) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (B) $-18\hat{i} - 13\hat{j} + 2\hat{k}$ (C) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (D) $6\hat{i} - 2\hat{j} + 8\hat{k}$

LEVEL # 2

Q.1 A particle is moving in a circle with uniform speed v. In moving from a point to another diametrically opposite point-

(A) The momentum changes by mv
(B) The momentum changes by 2mv
(C) The kinetic energy changes by (1/2) mv^2
(D) The kinetic energy changes by mv^2.

Q.2 In uniform circular motion-

(A) Both the angular velocity and the angular momentum vary.
(B) The angular velocity varies but the angular momentum remains constant.
(C) Both the angular velocity and the angular momentum stay constant.
(D) The angular momentum varies but the angular velocity remains constant.

Q.3 When a body moves in a circular path, no work is done by the force since-

(A) There is no displacement
(B) There is no net force
(C) Force and displacement are perpendicular to each other
(D) The force is always away from the centre.

Q.4 Which of the following statements is false for a particle moving in a circle with a constant angular speed-

(A) The velocity vector is tangent to the circle
(B) The acceleration vector is tangent to the circle
(C) The acceleration vector points to the centre of the circle
(D) The velocity and acceleration vectors are perpendicular to each other.

Q.5 If a_r and a_t represent radial and tangential accelerations, the motion of a particle will be uniformly circular if-

(A) $a_r = 0$ and $a_t = 0$
(B) $a_r = 0$ but $a_t \neq 0$
(C) $a_r \neq 0$ but $a_t = 0$
(D) $a_r \neq 0$ and $a_t \neq 0$

Q.6 If the radii of curvature of the path of two particles of same masses are in the ratio 1 : 2, then in order to have constant centripetal force, their velocity should be in the ratio of-

(A) 1 : 4
(B) 4 : 1
(C) $\sqrt{2}$: 1
(D) 1 : $\sqrt{2}$

Q.7 An object is moving in a circle of radius 100 m with a constant speed of 31.4 m/s. What is its average speed for one complete revolution-

(A) Zero
(B) 31.4 m/s
(C) 3.14 m/s
(D) $\sqrt{2} \times 31.4$ m/s

Q.8 A body of mass 1 kg tied to one end of string is revolved in a horizontal circle of radius 0.1 m with a speed of 3 revolutions/sec, assuming the effect of gravity is negligible, then linear velocity, acceleration and tension in the string will be-

(A) 1.88 m/s, 35.5 m/s^2, 35.5 N
(B) 2.88 m/s, 45.5 m/s^2, 45.5 N
(C) 3.88 m/s, 55.5 m/s^2, 55.5 N
(D) None of these

Q.9 A cyclist is riding with a speed of 27 km h^{-1}. As he approaches a circular turn on the road of radius 80 m, he applies brakes and reduces his speed at a constant rate of 0.5 ms^{-1}. The magnitude of the net acceleration of the cyclist is :

(A) 0.86 ms^{-2} (B) 0.43 ms^{-2} (C) 1.24 ms^{-2} (D) 1.76 ms^{-2}

Q.10 A 500 kg crane takes a turn of radius 50m with velocity of 36 km/hr. The centripetal force is-

(A) 1200 N (B) 1000 N (C) 750 N (D) 250 N

Q.11 Two bodies of equal masses revolve in circular orbits of radii R_1 and R_2 with the same period. Their centripetal forces are in the ratio-

(A) $\left(\frac{R_2}{R_1}\right)^2$ (B) $\frac{R_1}{R_2}$ (C) $\left(\frac{R_1}{R_2}\right)^2$ (D) $\sqrt{R_1R_2}$

Q.12 In case of uniform circular motion which of the following physical quantities does not remain constant-

(A) Speed (B) Momentum (C) Kinetic energy (D) Mass

Q.13 What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and half the angular velocity ω

(A) The centripetal acceleration remains unchanged

(B) The centripetal acceleration is halved

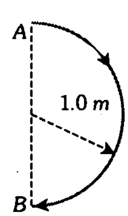
(C) The centripetal acceleration is doubled

(D) The centripetal acceleration is quadrupled

Q.14 A mass is supported on a frictionless horizontal surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω_0. If the length of the string and angular velocity are doubled, the tension in the string which was initially T_0 is now-

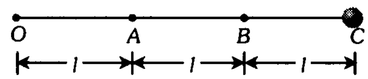
(A) T_0 (B) $T_0/2$ (C) $4T_0$ (D) $8T_0$

Q.15 In 1.0 s, a particle goes from point A to point B, moving in a semicircle of radius 1.0 m (see figure). The magnitude of the average velocity is-



(A) 3.14 m/s (B) 2.0 m/s (C) 1.0 m/s (D) Zero

Q.16 Three identical particles are joined together by a thread as shown in figure. All the three particles are moving in a horizontal plane. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is-



(A) 3 : 5 : 7 (B) 3 : 4 : 5 (C) 7 : 11 : 6 (D) 3 : 5 : 6

Q.17 A particle is moving in a circle of radius R with constant speed v. If radius is doubled, then its centripetal force to keep the same speed should be-

(A) Doubled (B) Halved (C) Quadrupled (D) Unchanged

Q.18 A stone tied to the end of a string 1m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 seconds, what is the magnitude and direction of acceleration of the stone-

(A) $\frac{\pi^2}{4}$ ms^{-2} and direction along the radius towards the centre.

(B) π^2 ms^{-2} and direction along the radius away from the centre.

(C) π^2 ms^{-2} and direction along the radius towards the centre.

(D) π^2 ms^{-2} and direction along the tangent to the circle.

Q.19 What is the angular velocity of earth-

(A) $\frac{2\pi}{86400}$ rad/sec (B) $\frac{2\pi}{3600}$ rad/sec (C) $\frac{2\pi}{24}$ rad/sec (D) $\frac{2\pi}{6400}$ rad/sec

Q.20 If the length of the second's hand in a stop clock is 3 cm, the angular velocity and linear velocity of the tip is-

(A) 0.2047 rad/sec., 0.0314 m/sec (B) 0.2547 rad/sec., 0.314 m/sec

(C) 0.1472 rad/sec., 0.06314 m/sec (D) 0.1047 rad/sec., 0.00314 m/sec.

Q.21 A coin, placed on a rotating turn-table slips when it is placed at a distance of 9 cm from the centre. If the angular velocity of the turn-table is tripled, it will just slip, if its distance from the centre is-

(A) 27 cm (B) 9 cm (C) 3 cm (D) 1 cm

Q.22 When a ceiling fan is switched off, its angular velocity reduces to 50% while it makes 36 rotations. How many more rotations will it make before coming to rest (Assume uniform angular retardation)

(A) 18 (B) 12 (C) 36 (D) 48

Q.23 For a particle in circular motion, the centripetal acceleration is-

(A) Less than its tangential acceleration

(B) Equal to its tangential acceleration

(C) More than its tangential acceleration

(D) May be more or less than its tangential acceleration

Q.24 A particle moves in a circular path with decreasing speed. Choose the correct statement-

(A) Angular momentum remains constant

(B) Acceleration ($\vec{a}$) is towards the centre

(C) Particle moves in a spiral path with decreasing radius

(D) The direction of angular momentum remains constant.

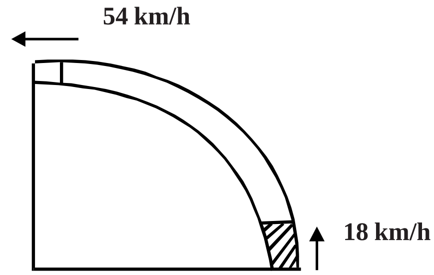
Q.25 The angle turned by a body undergoing circular motion depends on time as $\theta = \theta_0 + \theta_1 t + \theta_2 t^2$. Then the angular acceleration of the body is-

(A) θ_1 (B) θ_2 (C) $2\theta_1$ (D) $2\theta_2$.

LEVEL # 3

MORE THAN ONE CHOICE MAY BE CORRECT :

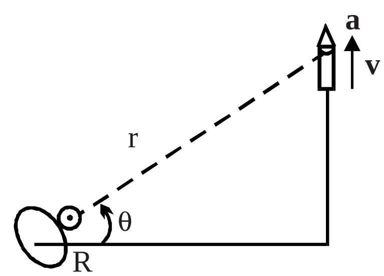
Q.1 A car enters a curved road in the form of a quarter of a circle, the path length being 200 m. Its speed at the entrance is 18 km/h but when it leaves, it increases to 54 km/h. If the car is travelling with constant acceleration along the curve, the acceleration when the car leaves the curved road is



(A) 0.92 m/s^2 (B) 0.54 m/s^2

(C) 1.84 m/s^2 (D) 2.76 m/s^2

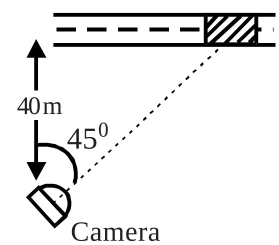
Q.2 A rocket is fired vertically and tracked by the radar R as shown in the figure. At a particular angular position $\theta = 60^0$, measured parameters are r = 9 km, $d^2r/dt^2 = 21\ m/s^2$ and $d\theta/dt = 0.02$ rad/s. The acceleration of the rocket at this position is



(A) 20 m/s^2 (B) 10 m/s^2

(C) 2 m/s^2 (D) 200 m/s^2

Q.3 A racing car is travelling along a track at a constant speed of 40 m/s. A television cameraman is recording the event from a distance of 40 m directly away from the track as shown in figure. In order to keep the car under view, the camera is rotated with a certain angular velocity. This angular velocity should be



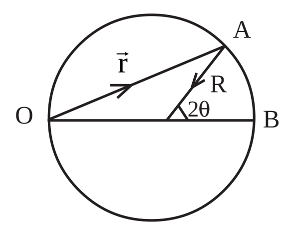
(A) 1.0 rad/s (B) 5 rad/s

(C) 0.5 rad/s (D) 2.0 rad/s

Q.4 The equation of motion of a particle moving on circular path (radius 200 m) is given by $s = 18t + 3t^2 - 2t^3$, where s is the total distance covered from straight point in metres at the end of t seconds. The maximum speed of the particle will be

(A) 15 m/sec (B) 23 m/sec (C) 19.5 m/sec (D) 25 m/sec.

Q.5 A particle 'A' moves along a circle of radius R = 1/2 m, so that its radius vector $\vec{r}$ relative to the point O rotates with the constant angular velocity $\omega = 0.4$ rad/s. The acceleration of the particle is



(A) $4\omega^2 R$ (B) $2\omega^2 R$

(C) $\omega^2 R$ (D) $3\omega^2 R$

Q.6 A particle is moving along a circular path. The angular velocity, velocity, angular acceleration and centripetal acceleration of the particle at any instant respectively are ω, v, α and a_c. Which of the following relations is/are correct ?

(A) $\vec{\omega} \perp \vec{v}$ (B) $\vec{\omega} \perp \vec{\alpha}$ (C) $\vec{\omega} \perp \vec{a}_c$ (D) $\vec{v} \perp \vec{a}_c$

Q.7 In hydrogen atom, the electron is moving round the nucleus with a velocity of 2.2×10^6 m/s in an orbit of radius 0.53 Å. The acceleration of the electron is

(A) 9×10^{18} m/s^2 (B) 9×10^{22} m/s^2 (C) 9×10^{-22} m/s^2 (D) 9×1012 m/s^2

Q.8 A toy car travels in a horizontal circle of radius 2a on the track by radial elastic spring of unstressed length a. The period of rotation of toy car is T_0. Now, the toy car is speeded up until it is moving in a circle of radius 4a. Assuming that spring obeys Hooke's law (i. e. F $\propto$ extension.), the new period T is given by

(A) $2\,T_0$ (B) $\sqrt{\frac{4}{3}}\,T_0$ (C) $\sqrt{\frac{2}{3}}\,T_0$ (D) $\sqrt{\frac{3}{2}}\,T_0$

Q.9 A mass of 2 kg is whirled in a horizontal circle by means of a string at an initial speed of 5 rpm. Keeping the radius constant, the tension in the string is doubled. The new speed is nearly :

(A) 14 rpm (B) 10 rpm (C) 7 rpm (D) 20 rpm

Q.10 A stone tied to the end of a 20 cm long string is whirled in a horizontal circle. If the centripetal acceleration is 9.8 ms^{-2}, its angular speed in radian/sec is :

(A) 7 (B) 14 (C) 49 (D) 98

Q.11 A particle is acted upon by a force of constant magnitude, which is always perpendicular to the velocity of the particle. The motion of particle takes place in a plane. It follows that

(A) its velocity is constant (B) its kinetic energy is constant
(C) its acceleration is constant (D) it moves along an elliptical path.

Q.12 A particle is moving in a circle of radius R in such a way that at any instant the normal and tangential components of its acceleration are equal. If its speed at t = 0 is v_0, the time taken to complete the first revolution is :

(A) $\frac{R}{v_0}$ (B) $\frac{R}{v_0}\left(1-e^{-2\pi}\right)$ (C) $\frac{R}{v_0}e^{-2\pi}$ (D) $\frac{2\pi R}{v_0}$

Q.13 The velocity of electron in the 1st orbit of radius 0.528 Å in hydrogen atom is 2.18×10^6 ms^{-1}. As compared to Q. No. 7, the acceleration of the electron is :

(A) Doubled (B) Halved (C) Unchanged (D) Quadrupled

Q.14 A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of the particle in metre per second^2 is :

(A) π^2 (B) $8\pi^2$ (C) $2\pi^2$ (D) $4\pi^2$

Q.15 The length of a second's hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is :

(A) Zero (B) $\frac{\pi}{30}\sqrt{2}$ cm / sec (C) $\frac{\pi}{30\sqrt{2}}$ cm/sec (D) $\frac{\pi}{30}$ cm / sec

Q.16 A toy-car travels in a horizontal circle of radius 2a, kept on the track by radial elastic string of unstressed length a. The period of rotation of the toy-car is T_0. Now the toy car is speeded up until it is moving in a circle of radius 3a. Assuming that the string obeys Hooke's law the new period T is given as :

(A) $T = T_0$ (B) $T = \frac{\sqrt{3}}{2}T_0$ (C) $T = \sqrt{\frac{4}{3}}T_0$ (D) $T = \frac{3}{4}T_0$

Q.17 A point moves along an arc of a circle of radius R. Its velocity depends upon the distance covered s as V = a$\sqrt{s}$, where a is constant. The angle θ between the vector of total acceleration and tangential acceleration is :

(A) $\tan\theta = \sqrt{\frac{s}{R}}$ (B) $\tan\theta = \sqrt{\frac{s}{2R}}$ (C) $\tan\theta = \frac{s}{2R}$ (D) $\tan\theta = \frac{2s}{R}$

Q.18 A car is travelling at 20 m/s on a circular road of radius 100 m. It is increasing its speed at the rate of $3 m/s^2$. Its acceleration is :

(A) $3 m/s^2$ (B) $4 ms^{-2}$ (C) $5 m/s^2$ (D) $7 m/s^2$

Q.19 A train moving with a constant speed along a straight track takes a bend in a curve with the same speed. Due to this

(A) Only magnitude of velocity is changed
(B) Speed is changed
(C) Only direction of velocity is changed
(D) Velocity is not changed

Q.20 A particle covers 1/4th of a circle of radius r, then the distance traversed and the displacement are respectively

(A) $1/2\ \pi r$; r (B) $1/2\pi r$; 2r (C) $1/2\pi r$; r/2 (D) $1/2\pi r$; $r\sqrt{2}$

Q.21 The rotation of a rigid body is defined by

$$\varphi = t^3 - 1.5t^2$$

The angular velocity for t = 2 sec., will be :

(A) $2\ s^{-1}$ (B) $4\ s^{-1}$ (C) $6\ s^{-1}$ (D) $8\ s^{-1}$

Q.22 The rotation of a rigid body is defined by

$$\varphi = 2.5\sin\frac{\pi t}{4}$$

The angular velocity for t = 2 sec. will be

(A) $\frac{\pi}{4}\sec^{-1}$ (B) $\frac{\pi}{8}\sec^{-1}$ (C) $\frac{\pi}{10}\sec^{-1}$ (D) Zero

Q.23 A proton goes round in a circular orbit of radius 10 cm under a centripetal force of 4×10^{-13} newton. Calculate the frequency of revolution of the proton (assume $m_p = 1.6 \times 10^{-27}$ kg)

(A) 4×10^9 Hz (B) 16×10^8 Hz (C) 8×10^6 Hz (D) 2.5×10^6 Hz

Q.24 A particle is moving along a circular path of radius 5m with uniform speed $5ms^{-1}$. What will be the average acceleration when the particle completed half revolution ?

(A) 0 (B) 10 ms^{-2} (C) 10π ms^{-2} (D) $10/\pi$ ms^{-2}

ANSWER KEY

LEVEL # 1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	C	C	B	A	C	A	B	C	B	C	C	C	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	B	B	C	B	B	C	C	D	B	A	B	C	C
Que.	31	32	33	34	35	36	37								
Ans.	D	B	D	A	A	C	B								

LEVEL # 2

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	C	B	C	D	B	A	A	B	B	B	A	D	B
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	D	B	C	A	D	D	B	D	D	D					

LEVEL # 3

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	A	C	C	A	A,C,D	B	C	C	A	B	B	C	D	B
Que.	16	17	18	19	20	21	22	23	24						
Ans.	B	D	C	C	D	C	D	C	D						