

GRAPHICAL INTERPRETATION

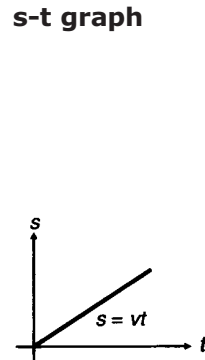
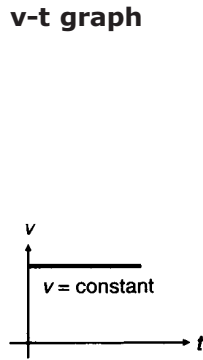
GRAPHICAL INTERPRETATION

CONCEPTS FOR GRAPHS

TABLE : SIMPLE (S - T) AND (V - T) GRAPHS FOR DIFFERENT TYPES OF MOTION

S.N. Different Cases

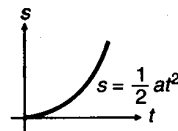
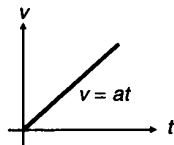
1. Uniform motion
(with $s = 0$ at $t = 0$)



Important Points

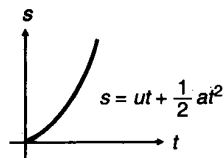
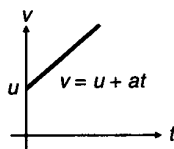
- (i) $v = \text{constant}$
- (ii) Slope of s-t graph
= $v = \text{constant}$.

2. Uniformly accelerated motion with $u = 0$ and $s = 0$ at $t = 0$



- (i) $u = 0$, i.e., slope of s-t graph at $t = 0$, should be zero.
- (iii) a or slope of v-t graph is constant.

3. Uniformly accelerated motion with $u \neq 0$ but $s = 0$ at $t = 0$

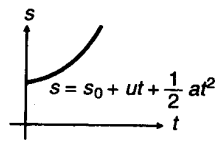
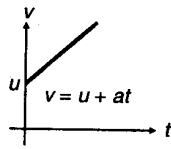


- (i) $u \neq 0$, i.e., v or slope of s-t graph at $t = 0$ is not zero.
- (ii) v or slope of s-t graph gradually goes on increasing.

4. Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$

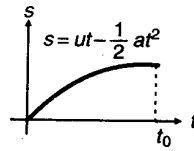
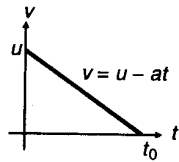
- (i) $s = s_0$ at $t = 0$.

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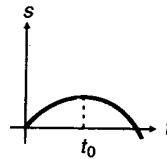
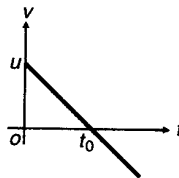
5. Uniformly retarded motion till velocity becomes zero

- (i) Slope of s-t graph at $t = 0$ gives u .
- (ii) Slope of s-t graph at $t = t_0$ becomes zero.
- (iii) In this case, u can't be zero.

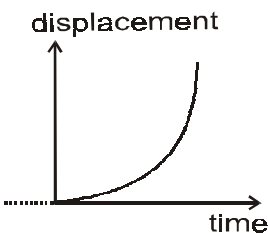


6. Uniformly retarded then accelerated in opposite direction.

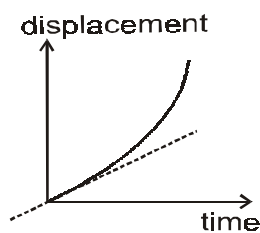
- (i) At time $t = t_0$, $v = 0$ or slope of s-t graph is zero.
- (ii) In s-t graph, slope or velocity first decreases, then increases with opposite sign.



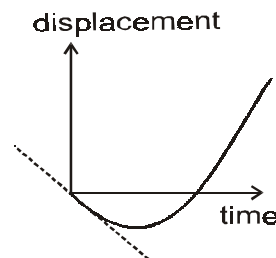
7. Positively accelerated motion. The graph is upward opening parabola.



$$x_0 = 0, v_0 = 0, a > 0$$



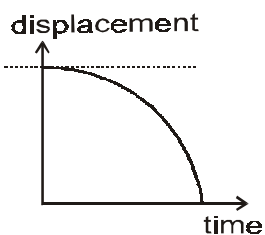
$$x_0 = 0, v_0 > 0, a > 0$$



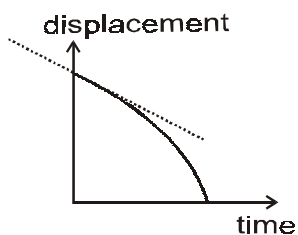
$$x_0 = 0, v_0 < 0, a > 0$$

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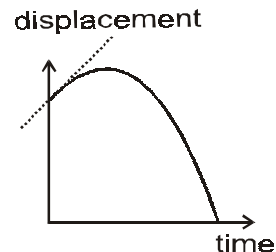
8. Negatively accelerated motion. The graph is downward opening parabola.



$$x_0 \neq 0, v_0 = 0, a < 0$$



$$x_0 \neq 0, v_0 < 0, a < 0$$



$$x_0 \neq 0, v_0 > 0, a < 0$$

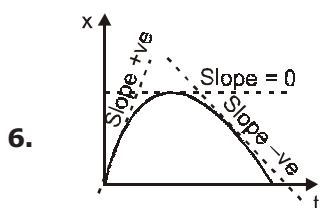
NOTE This topic will be dealt in detail in next chapter : motion in one dimension and motion under gravity

Displacement-time graph, practically can't take sharp turn because it will then mean two or more velocities at a time (at peak) while (v - t) graph, to some extent can have sharp curve as two different accelerations at a single instant are possible. Sudden strokes of forces in different directions can be applied at an instant.

(Notation : (x - t) \Rightarrow displacement time graph, (v - t) \Rightarrow velocity time graph, (a - t) \Rightarrow acceleration time graph.)

CONCEPTS REGARDING SLOPES IN GRAPHS

1. Slope in displacement time-graph at any time gives instantaneous velocity at that time.
2. Tangent at origin in (x - t) graph gives direction of initial velocity.
3. Slope in velocity time-graph at any time gives instantaneous acceleration at that time.
4. Tangent at origin in (v - t) graph gives direction of initial acceleration.
5. Slope in (v - x) graph at any instant gives ratio of $\left(\frac{a}{v}\right)$ at that instant because $\frac{a}{v} = \frac{dv}{dx}$



In the above graph

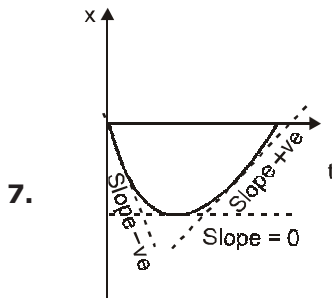
Slope = $\frac{dx}{dt}$ is tending from positive towards negative, so slope is decreasing with time. So rate of

change of slope i.e. $\frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2x}{dt^2}$ is negative i.e. $\frac{d^2x}{dt^2} < 0$, which is the condition of maxima, so it's

a downward opening parabola or hill-structure. As acceleration $a = \frac{d^2x}{dt^2}$.

\therefore here $a < 0$

GRAPHICAL INTERPRETATION

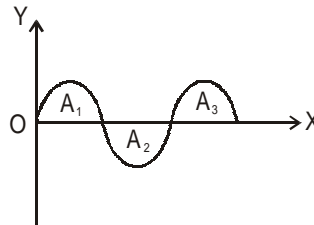


∴ Slope = $\frac{dx}{dt}$ increasing with times so $\frac{d^2x}{dt^2} > 0$ which is condition of minima so it's upward opening parabola or valley - structure.

Here $a = \frac{d^2x}{dt^2} > 0$, so it's showing +vely accelerated motion

CONCEPTS REGARDING AREA IN GRAPH

8. Area : In graphs, positive and negative area should be treated differently i.e. A_1 and A_3 are positive and A_2 negative



9. Area under (v - t) graph gives displacement during that time (Area to be taken with sign).

$$\text{i.e. } \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v dt$$

eg. Sum of areas with sign gives displacement while without sign gives distance.

10. Area under (a - t) graph gives change in velocity during that time (Area to be taken with sign).

$$\text{i.e. } \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a dt$$

11. Area under (a - x) graph gives $\frac{(v_2^2 - v_1^2)}{2}$ for time $t_2 - t_1$.

$$\int_{v_1}^{v_2} v dv = \int_{x_1}^{x_2} a dx$$

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12. Area under (a - x) graph gives $\frac{v^2}{2}$, if initial velocity is zero.

<< method of obtaining one type of graph if another type of graph is given >>

13. (v - t) graph can be obtained from (x - t) graph by differentiating

$$\frac{dx(t)}{dt} = v(t) \quad \text{i.e. } x(t) \xrightarrow{d} v(t)$$

14. (a - t) graph can be obtained from (v - t) graph by differentiating

$$\frac{dv(t)}{dt} = a(t) \quad \text{i.e. } v(t) \xrightarrow{d} a(t)$$

- Ex. If (v - t) graph is linear, then it means that (a - t) is constant

15. (a - t) graph can be obtained from (x - t) graph by double differentiating

$$a(t) = \frac{d^2x(t)}{dt^2} \quad \text{i.e. } x(t) \xrightarrow{dd} a(t)$$

16. (a - x) graph can be obtained from (v - x) graph by differentiating

$$a(x) = v(x) \frac{dv}{dx} \quad \text{i.e. } v(x) \xrightarrow{d} a(x)$$

- Ex. If (v - x) graph is linear, then it does not mean (a - x) is constant graph.

17. (x - t) graph can be obtained from (v - t) graph by integrating

$$\int v(t) dt = \int dx = [x]_i^f \quad \text{i.e. } v(t) \xrightarrow{i} x(t)$$

18. (v - t) graph can be obtained from (a - t) graph by integrating

$$\int a dt = \int dv = [v]_i^f \quad \text{i.e. } a(t) \xrightarrow{i} v(t)$$

19. (v - x) graph can be obtained from (a - x) graph by integrating

$$\int a dx = \int v dv = \left[\frac{v^2}{2} \right]_i^f \quad \text{i.e. } a(x) \xrightarrow{i} v(x)$$

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20. $(x - t)$ graph can be obtained from $(a - t)$ graph by double integrating.

$$\int \left[\int [adt] \right] dt = \int \left[\int [dv] \right] dt = \int dx = [x]_i^f \quad \text{i.e. } a(t) \xrightarrow{\text{ii}} x(t)$$

21. $(x - t)$ graph can be obtained from $(v - x)$ graph by integrating.

$$\int \frac{dv}{v(x)} = \int dt = [t]_i^f \quad \text{i.e. } v(x) \xrightarrow{\text{i}} x(t)$$

22. $(v - t)$ graph can be obtained from $(a - v)$ graph by integrating

$$\int \frac{dv}{a(v)} = \int dt = [t]_i^f \quad \text{i.e. } a(v) \xrightarrow{\text{i}} v(t)$$

23. $(v - x)$ graph can be obtained from $(a - v)$ graph by integrating

$$\int \frac{v dv}{a(v)} = \int dx = [x]_i^f \quad \text{i.e. } a(v) \xrightarrow{\text{i}} v(x)$$

24. $(x - t)$ graph can be obtained from $(a - x)$ graph by double integrating

$$\int \left[\int \left[\frac{dx}{a(x)} \right]_i^f dx \right]_i^f = \int \left[\int dt \right] dt = \left[\frac{t^2}{2} \right]_i^f \quad \text{i.e. } a(x) \xrightarrow{\text{ii}} x(t)$$

Ex. $\int \left[\int dt \right] dt = \int \left[\int \frac{dx}{a(x)} \right] dx$

$$a(x) \longrightarrow x(t) \quad (\text{at } t = 0 \longrightarrow x = 0)$$

$$\text{at } t = t \longrightarrow x = x)$$

$$a = b x^2$$

$$\int \frac{dx}{a(x)} = \int \frac{dx}{bx^2} = -\frac{1}{bx}$$

$$\int \left[\frac{-1}{bx} \right] dx = \left[\frac{-1}{b} \log x \right]_i^f = \left[\frac{t^2}{2} \right]_i^f$$

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25. $(v - x)$ graph can be obtained from $(x - t)$ graph by differentiating $x(t)$ and substituting $x(t) \xrightarrow{d.s} v(x)$

EXAMPLE :

$$x = b^2 t^2 \quad \therefore t = \frac{\sqrt{x}}{b} \quad (t > 0)$$



$$\frac{dx}{dt} = 2b^2 t = 2b^2 \frac{\sqrt{x}}{b} = 2b\sqrt{x}$$

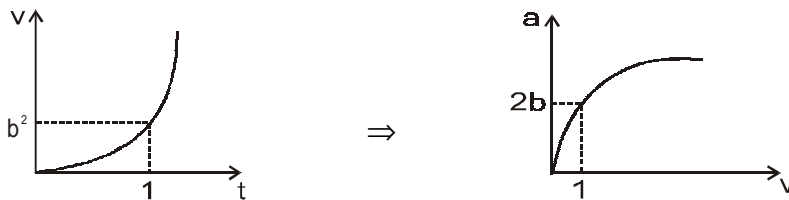
$$v = \frac{dx}{dt} = 2b\sqrt{x}$$

so $v = 2b\sqrt{x}$



26. $(a - v)$ graph can be obtained from $(v - t)$ graph by differentiating $v(t)$ and substituting $v(t) \xrightarrow{d.s} a(v)$

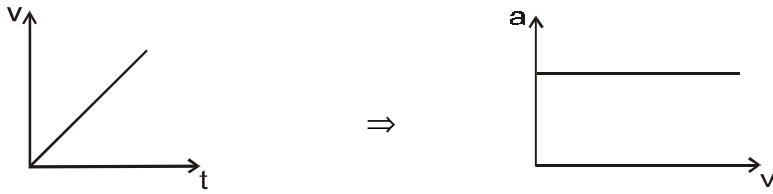
Ex. $v = b^2 t^2, \quad t = \frac{\sqrt{v}}{b} \quad (t > 0)$



$$\frac{dv}{dt} = 2b^2 t = 2b^2 \frac{\sqrt{v}}{b} = 2b\sqrt{v}$$

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$$a = \frac{dv}{dt} = 2b\sqrt{v} \text{ so } a = 2b\sqrt{v}$$



- 27.** (a - v) graph can be obtained from (v - x) graph by differentiating v (x) and substituting v (x) $\xrightarrow{d.s}$ a (v)

Ex. $v = b^2x^2 \Rightarrow x = \pm \frac{\sqrt{v}}{b}$ given ($x > 0$)

$$\frac{dv}{dx} = 2b^2x$$

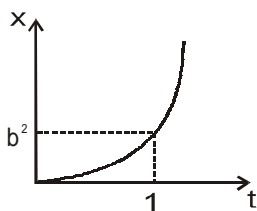
$$= \pm \frac{2b^2\sqrt{v}}{b}$$

$$\frac{dv}{dx} = +2b\sqrt{v}$$

$$a = v \frac{dv}{dx} = v2b\sqrt{v} = 2bv^{3/2}$$

- 28.** (a - x) graph can be obtained from (x - t) graph by differentiating x (t) & substituting x (t) \longrightarrow a (x) x (t) \longrightarrow v (x) \longrightarrow a (x)

Ex.1 $x = b^2t^2$ so $t = \frac{\sqrt{x}}{b}$ $t > 0$ only

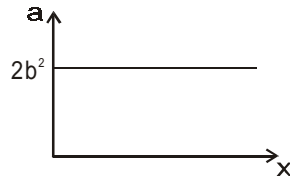


$$v = \frac{dx}{dt} = 2b^2t = 2b^2 \frac{\sqrt{x}}{b} = 2b\sqrt{x}$$

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$$\frac{dv}{dx} = 2b \frac{1}{2} x^{-1/2} = \frac{b}{\sqrt{x}}$$

$$a = v \frac{dv}{dx} = 2b \sqrt{x} \cdot \frac{b}{\sqrt{x}} = 2b^2 \quad (a = \text{const})$$



Ex.2 $x = b^3 t^3$ or $t = \left[\frac{x}{b^3} \right]^{1/3} = \frac{x^{1/3}}{b}$

$$v = \frac{dx}{dt} = 3b^3 t^2 = 3b^3 \cdot \frac{x^{2/3}}{b^2} = 3bx^{2/3}$$

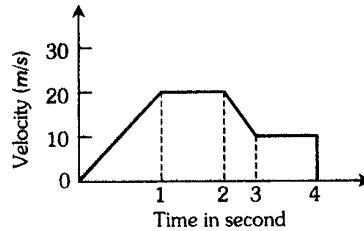
$$\therefore \frac{dv}{dx} = 3b \cdot \frac{2}{3} x^{-1/3} = 2bx^{-1/3}$$

$$a = v \frac{dv}{dx} = 3b x^{2/3} \cdot 2bx^{-1/3} = 6b^2 x^{1/3}$$

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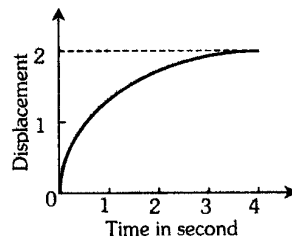
PRACTICE QUESTIONS BASED ON GRAPHS

Q.1 The variation of velocity of a particle with time moving along a straight line is illustrated in the following figure. The distance travelled by the particle in four seconds is-



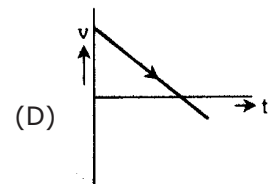
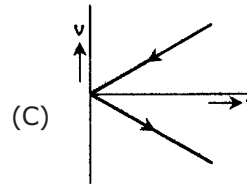
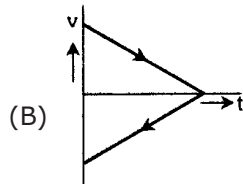
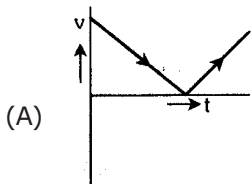
- (A) 60m (B) 55m (C) 25m (D) 30m

Q.2 The displacement of a particle as a function of time is shown in the figure. The figure shows that-



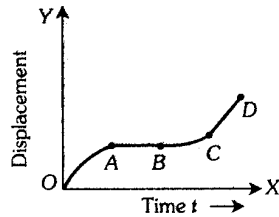
- (A) The particle starts with certain velocity but the motion is retarded and finally the particle stops.
(B) The velocity of the particle is constant throughout.
(C) The acceleration of the particle is constant throughout.
(D) The particle starts with constant velocity, then motion is accelerated and finally the particle moves with another constant velocity.

Q.3 A ball is thrown vertically upwards. Which of the following graph/graphs represent velocity-time graph of the ball during its flight (air resistance is neglected)



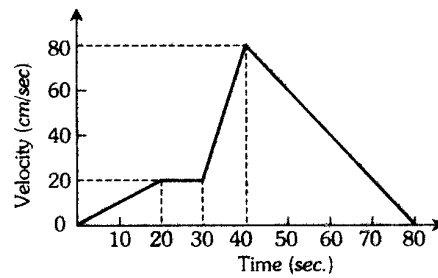
GRAPHICAL INTERPRETATION

- Q.4** The graph between the displacement x and time t for a particle moving in a straight line is shown in figure. During the interval OA, AB, BC and CD, the acceleration of the particle is-



- OA, AB, BC, CD OA, AB, BC, CD OA, AB, BC, CD OA, AB, BC, CD
 (A) + 0 + + (B) - 0 + 0 (C) + 0 - + (D) - 0 - 0

- Q.5** The v-t graph of a moving object is given in figure. The maximum acceleration is-



- (A) 1 cm/sec² (B) 2 cm/sec² (C) 3 cm/sec² (D) 6 cm/sec²

ANSWERS :

1. (B) 2. (A) 3. (D) 4. (B) 5. (D)

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EXAMPLES BASED ON GRAPHS

Ex.1 A particle is moving in a plane with velocity given by:

$$\vec{v} = \hat{i}u_0 + \hat{j}a\omega\cos\omega t$$

If the particle is at the origin at $t = 0$; (a) Calculate the trajectory of the particle; (b) Find its distance from the origin at time $(3\pi/2\omega)$.

Sol.

$$\vec{v} = \hat{i}v_x + \hat{j}v_y$$

where $v_x = u_0$ and $v_y = a\omega\cos\omega t$

i.e.,
$$\frac{dx}{dt} = u_0 \text{ and } \frac{dy}{dt} = a\omega\cos\omega t$$

which on integration gives, $x = \int u_0 dt$ and $y = \int a\omega\cos\omega t dt$

i.e.,
$$x = u_0 t + A \quad \text{and} \quad y = a \sin \omega t + B$$

Now, at $t = 0, x = 0$ and $y = 0$

so $A = 0$ and $B = 0$

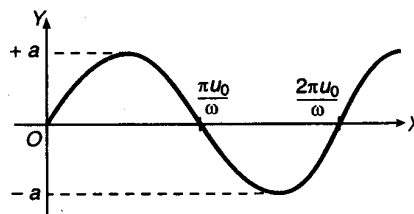
$\therefore x = u_0 t$ (i)

and $y = a \sin \omega t$ (ii)

(a) Substituting the value of t from Eqn. (i) in (ii),

$$y = a \sin (\omega x / u_0)$$

This is the desired trajectory and it is a sine curve as shown in Fig. 2.72.



(b) For $t = (3\pi/2\omega)$ from Eqs. (i) and (ii), we have

$$x = u_0 (3\pi/2\omega) \quad \text{and} \quad y = -a$$

So, distance from the origin at $t = (3\pi/2\omega)$ will be

$$r = \sqrt{x^2 + y^2} = \sqrt{[3\pi u_0 / 2\omega]^2 + a^2} \quad \text{ANS.}$$

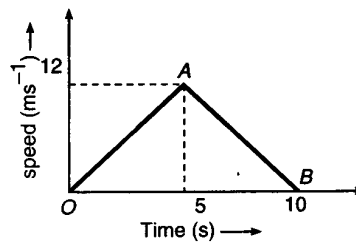
GRAPHICAL INTERPRETATION

Ex.2 The speed-time graph of a particle moving along a fixed direction is shown in Fig. 2.73. Obtain the distance travelled by the particle between (i) $t = 0$ to 10 s (ii) $t = 2$ to 6 s. What is the average speed of the particle in intervals in (i) and (ii)?

Sol. (i) Distance travelled by the particle between $t = 0$ to 10 s is given by

$$s = \text{Area of } \triangle OBA = \frac{1}{2} \times 10 \times 12 = 60 \text{ m Ans.}$$

$$\begin{aligned} \text{Average speed} &= \frac{\text{total distance covered}}{\text{total time taken}} = \frac{60}{10} \\ &= 6 \text{ ms}^{-1} \qquad \qquad \qquad \text{Ans.} \end{aligned}$$



(ii) Acceleration of the particle during journey OA is given by

$$v = u + at \quad \text{or} \quad 12 = 0 + 5a$$

$$\text{or} \quad a = +2.4 \text{ ms}^{-2}$$

Similarly, acceleration of the particle during journey AB is given by

$$v = u + at \quad \text{or} \quad 0 = 12 + 5a$$

$$\text{or} \quad a = -2.4 \text{ ms}^{-2}$$

Velocity of the particle after 2 s from start will be

$$v = u + at = 0 + 2.4 \times 2 = 4.8 \text{ ms}^{-1}$$

\therefore Distance covered by the particle between $t = 2$ to 5 s (in 3 s) is given by

$$s_1 = ut + \frac{1}{2} at^2 = 4.8 \times 3 + \frac{1}{2} \times 2.4 \times 3^2 = 25.2 \text{ m}$$

Distance covered by the particle in $t = 5$ to 6 s (in 1 s) is given by

$$s_2 = ut + \frac{1}{2} at^2 = 12 \times 1 + \frac{1}{2} \times (-2.4) \times 1^2 = 10.8 \text{ m}$$

Total distance travelled in $t = 2$ to 6 s,

$$s = s_1 + s_2 = 25.2 + 10.8 = 36 \text{ m}$$

Average speed in the interval ($t = 2$ to 6 s)

$$= \frac{\text{total distance covered}}{\text{total time taken}} = \frac{36}{4} = 9 \text{ ms}^{-1}. \qquad \qquad \qquad \text{Ans.}$$

GRAPHICAL INTERPRETATION

Ex.3 A ball is thrown upward with an initial velocity of 100 ms^{-1} . After how much time will it return? Draw velocity-time graph for the ball and find from the graph (i) the maximum height attained by the ball and (ii) height of the ball after 15 s. Take $g = 10 \text{ ms}^{-2}$.

Sol. Here, $u = 100 \text{ ms}^{-1}$, $g = -10 \text{ ms}^{-2}$

At highest point, $v = 0$

As $v = u + gt \therefore 0 = 100 - 10 \times t$

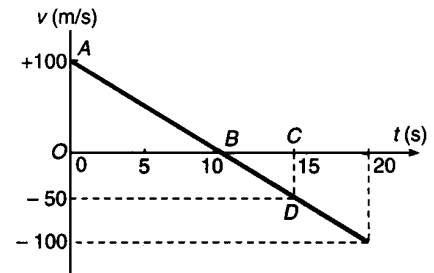
\therefore Time taken to reach highest point,

$$t = \frac{100}{10} = 10 \text{ s}$$

\therefore The ball will return to the ground at $t = 20 \text{ s}$.

Corresponding velocity-time graph of the ball is shown in

Fig.



(i) Maximum height attained by the ball = Area of $\triangle AOB$

$$= \frac{1}{2} \times 10 \times 100 = 500 \text{ m.}$$

Ans.

(ii) Height attained after 15 s = Area of $\triangle AOB$ + Area of $\triangle BCD$

$$= 500 + \frac{1}{2} (15 - 10) \times (-50) = 500 - 125 = 375 \text{ m.}$$

Ans.

Ex.4 A ball is dropped from a height of 80 m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Draw the speed-time graph of its motion between $t = 0$ to 11.2s.

[Take $g = 10 \text{ ms}^{-2}$]

Sol. (i) Time taken by the ball of fall through a height of 80 m is obtained as follows:

$$u = 0$$

$$h = \frac{1}{2} gt^2$$

$$80 = \frac{1}{2} \times 10 \times t^2 \text{ or}$$

$$t = \sqrt{\frac{2 \times 80}{10}} = 4.0 \text{ s}$$

Now,

$$v(t) = gt$$

\therefore

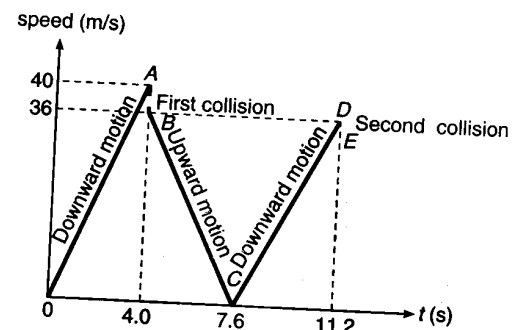
$$v(4) = 10 \times 4 = 40 \text{ ms}^{-1}$$

From time

$$t = 0 \text{ to } t = 4.0 \text{ s, } v(t) = gt = 10t,$$

or

$$v(t) \propto t$$



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In this duration, speed increases linearly with time t from 0 to 40 ms^{-1} during the downward motion of the ball and this speed-time variation has been shown by straight line OA in Fig.

- (ii) At first collision with the floor, speed lost by ball = $\frac{1}{10} \times 40 = 4.0 \text{ ms}^{-1}$.

Thus, the ball rebounds with a speed of $40 - 4 = 36.0 \text{ ms}^{-1}$. For the further upward motion, the speed at any instant t is given by

$$v(t) = v(0) - gt = 36 - 10 \times t$$

Now, the speed decreases linearly with time and becomes zero after time

$$t = \frac{36}{10} = 3.6 \text{ s}$$

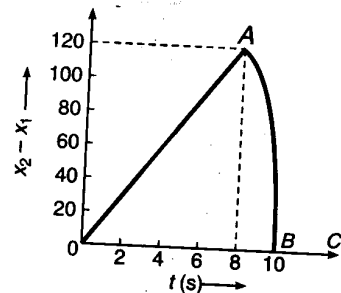
Thus, the ball reaches the highest point again after time $t = 4 + 3.6 = 7.6 \text{ s}$ from the start. Straight line BC represents the speed-time graph for this upward motion.

- (iii) At highest point, speed of ball is zero. It again starts falling. At any instant t , its speed is given by

$$v(t) = 10 t$$

Again, the speed of the ball increases linearly with time t from 0 to 36 ms^{-1} (initial speed of the previous upward motion) in the next time-interval of 3.6 s. Total time taken from the start = $4.0 + 3.6 + 3.6 = 11.2 \text{ s}$. This part of motion has been shown by straight line CD.

Ex.5 Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1} . Verify that the given graph correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take $g = 10 \text{ ms}^{-2}$. Give the equations for the linear and curved parts of the plot.



Sol.

$$x(t) = x_0 + ut + \frac{1}{2}gt^2$$

If we take origin for position measurement on the ground, then the positions of the two stones at any instant t will be

$$x_1 = 200 + 15t - \frac{1}{2} \times 10 \times t^2 \quad \dots(i)$$

$$x_2 = 200 + 30t - \frac{1}{2} \times 10 \times t^2 \quad \dots(ii)$$

When the first stone hits the ground, $x_1 = 0$

$$\text{or} \quad 200 + 15t - 5t^2 = 0 \quad \text{or} \quad t^2 - 3t - 40 = 0$$

$$\therefore t = \frac{3 \pm \sqrt{9+160}}{2} = \frac{3 \pm 13}{2} = 8\text{s} \quad \text{or} \quad -5 \text{ s}$$

As time cannot be negative, so $t = 8 \text{ s}$

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i.e., the first stone hits the ground after 8 s.

From Eqs. (i) and (ii), the relative position of second stone with respect to first is given by

$$x_2 - x_1 = 15 t$$

At $t = 8 \text{ s}$, $x_2 - x_1 = 15 \times 8 = 120 \text{ m}$

i.e., the separation between the stones is 120 m, when the first stone hits the ground.

As there is linear relationship between $x_2 - x_1$ and t , so the graph is straight line OA upto $t = 8 \text{ s}$. After $t = 8 \text{ s}$, only the second stone is in motion. So the graph is parabolic (AB) in accordance with quadratic equation,

$$x_2 = 200 + 30t - 5t^2$$

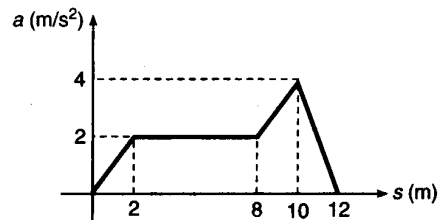
The second will hit the ground, when $x_2 = 0$

or $200 + 30t - 5t^2 = 0$

On solving, $t = 10 \text{ s}$

After $t = 10 \text{ s}$, the separation between the ball is zero, which explains the part BC of the graph.

Ex.6 The acceleration-displacement graph of a particle moving in a straight line is as shown alongside. Initial velocity of particle is zero. Find the velocity of the particle when displacement of the particle is, $s = 12 \text{ m}$.



Sol. We know that

$$v \frac{dv}{ds} = a \quad \text{or} \quad v dv = a ds \quad \text{or} \quad \int_0^v v dv = \int_0^s a ds$$

or $\frac{v^2}{2} = \text{area under } a - s \text{ graph} \quad \text{or} \quad v = \sqrt{2(\text{area under } a - s \text{ graph})}$

$$\begin{aligned} \text{area} &= \frac{1}{2} (2) (2) + 6 \times 2 + \frac{1}{2} (2 + 4) (2) + \frac{1}{2} \times 2 \times 4 \\ &= 2 + 12 + 6 + 4 = 24 \text{ m/s}^2 \end{aligned}$$

$\therefore v = \sqrt{2 \times 24} = 4\sqrt{3} \text{ m/s}$ Ans.

Ex.7 A car accelerates from rest at the rate of 1 m/s^2 for 5 seconds and then retards at the same rate till it comes to rest. Draw the $x-t$, $v-t$, and $a-t$ graph.

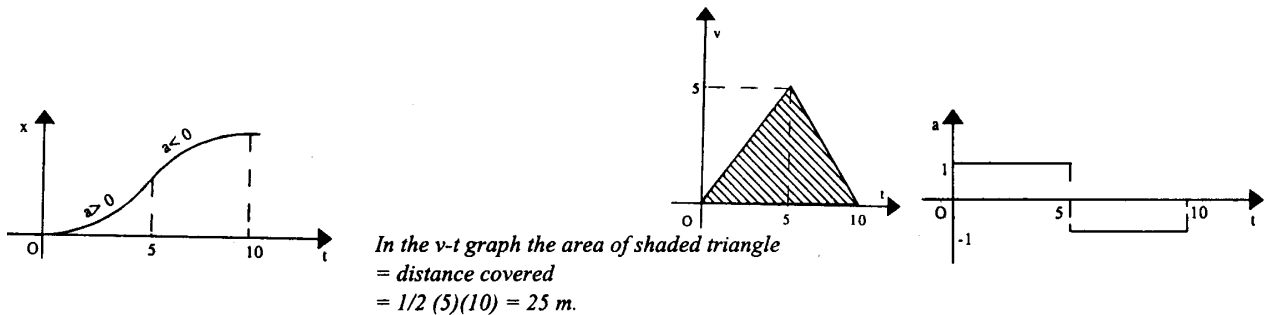
Sol. Velocity acquired after 5 sec. = $u + at = 0 + 1 (5) = 5 \text{ m/s}$

Time taken to come to rest = $\frac{0 - 5}{-1} = 5 \text{ sec.}$

\Rightarrow Car starts at $t = 0$ and accelerates till $t = 5 \text{ s}$.

The car starts slowing down at $t = 5 \text{ s}$ and comes to rest at $t = 10 \text{ s}$.

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Ex.8 A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β , to come to rest. If the total time elapsed is t sec. Evaluate (a) the maximum velocity reached and (b) the total distance travelled.

Sol. (a) Let the car accelerates for time t_1 and decelerates for time t_2 . Then

$$t = t_1 + t_2 \quad \dots (i)$$

and corresponding velocity-time graph will be as shown in Fig.

From the graph

$$\alpha = \text{slope of line OA} = \frac{v_{\max}}{t_1}$$

or $t_1 = \frac{v_{\max}}{\alpha} \quad \dots (ii)$

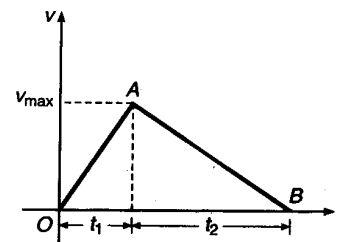
and $\beta = -\text{slope of line AB} = \frac{v_{\max}}{t_2}$

or $t_2 = \frac{v_{\max}}{\beta} \quad \dots (iii)$

From Eqs. (i), (ii) and (iii), we get

$$\frac{v_{\max}}{\alpha} + \frac{v_{\max}}{\beta} = t \quad \text{or} \quad v_{\max} \left(\frac{\alpha + \beta}{\alpha\beta} \right) = t$$

or $v_{\max} = \frac{\alpha\beta t}{\alpha + \beta}$



(b) Total distance = displacement
= area under v-t graph
= $\frac{1}{2} \times t \times v_{\max}$

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$$= \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha + \beta}$$

or Distance = $\frac{1}{2} \left(\frac{\alpha\beta t^2}{\alpha + \beta} \right)$

NOTE

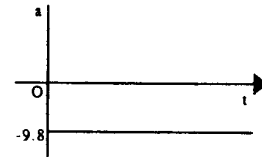
This problem can be solved by using equations of motion ($v = u + at$, etc.). Try it yourself.

Ex.9 A ball is thrown vertically upwards with a speed of 9.8 m/s from the ground. Draw the x-t, v-t and a-t graph for its motion.

Sol. As the acceleration of the ball remains $a = -9.8 \text{ m/s}^2$ throughout the motion, a-t graph is a horizontal line.

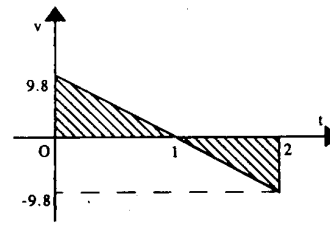
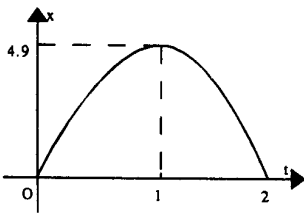
time taken by the ball to reach top

$$= \frac{0 - 9.8}{-9.8} = 1 \text{ sec}$$



and the height attained = $\frac{0^2 - 9.8^2}{2(-9.8)} = 4.9 \text{ m}$

As $a < 0$, x-t graph is a parabola opening downwards. The x-coordinate of the ball is zero initially and finally, when it reaches back after 2 seconds.



NOTE

(i) Velocity at the top = 0

Slope of the tangent to the x-t graph at $t = 1$ is zero.

(ii) Distance covered = Shaded area (taking area above or below t-axis as positive)

$$= 2 \left(\frac{1}{2} \times 1 \times 9.8 \right) = 9.8 \text{ m.}$$

$$\text{Displacement} = \text{Area (taking area below t-axis as negative)} = 4.9 - (4.9) = 0 \text{ m.}$$

(iii) Slope of line in v-t graph = Acceleration

$$= -9.8 \text{ m/s}^2$$

(iv) Between $t = 1$ & $t = 2$, velocity is -ve i.e., the ball is coming down.

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Ex.10 Velocity-time graph of a particle moving in a straight line is shown in Fig.

Plot the corresponding displacement-time graph of the particle if, at time $t = 0$, displacement $s = 0$.

Sol. Displacement = Area under velocity-time graph

$$\text{Hence, } s_{OA} = \frac{1}{2} \times 2 \times 10 = 10 \text{ m}$$

$$s_{AB} = 2 \times 10 = 20 \text{ m}$$

$$\text{or } s_{OAB} = 10 + 20 = 30 \text{ m}$$

$$s_{BC} = \frac{1}{2} \times 2 (10 + 20) = 30 \text{ m}$$

$$\text{or } s_{OABC} = 30 + 30 = 60 \text{ m}$$

$$\text{and } s_{CD} = \frac{1}{2} \times 2 \times 20 = 20 \text{ m}$$

$$\text{or } s_{OABCD} = 60 + 20 = 80 \text{ m}$$

Between 0 to 2s and 4 to 6s, motion is accelerated, hence displacement-time graph is a parabola. Between 2 to 4s, motion is uniform, so displacement-time graph will be a straight line. Between 6 to 8s, motion is decelerated, hence displacement-time graph is again a parabola but inverted in shape. At the end of 8s velocity is zero, therefore, slope of displacement-time graph should be zero. The corresponding graph is shown in Fig.

