PROJECTILE MOTION

PROJECTION FROM A HEIGHT

CASE (I)

HORIZONTAL PROJECTION (PROJECTILE THROWN PARALLEL TO THE HORIZONTAL)

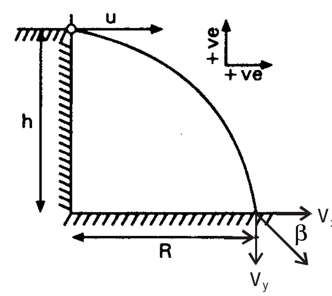
$u_x = u$

$u_y = 0$

$a_y = -g$

Horizontal motion $x = ut$ .... (1)

Vertical motion $-h = 0\,(t) - \frac{1}{2}gt^2$ .... (2)



From Eqn. (1) and (2)

$\Rightarrow$ $t = \sqrt{\frac{2h}{g}}$

Horizontal range (R) $= u_x t = u\sqrt{\frac{2h}{g}}$

(A) TRAJECTORY EQUATION

The path traced by projectile is called the trajectory.

After time t,

$x = ut$ ...(1)

$y = \frac{-1}{2}gt^2$ ....(2)

From eqn. (1) $t = x/u$

Put value of t in eqn. (2)

$$y = \frac{-1}{2}g.\frac{x^2}{u^2}$$

This is called trajectory equation

(B) VELOCITY AT A GENERAL POINT P (x,y)

$$v = \sqrt{u_x^2 + u_y^2}$$

Here, horizontal velocity of the projectile after time t

$v_x = u$

velocity of projectile in vertical direction after time t

$v_y = 0 + (-g)t = -gt = gt$ (downward)

$\therefore$ $v = \sqrt{u^2 + g^2t^2}$

and $\tan\theta = v_y/v_x$

PROJECTILE MOTION

Ex.1 *A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with a speed of 15 ms^{-1}. The vertical component of the velocity on hitting the ground is :*

(A) 79 ms^{-1} *(B) 89 ms^{-1}* *(C) 98 ms^{-1}* *(D) 108 ms^{-1}*

Sol. (C)

Ex.2 *In the previous question, the speed with which stone hits the ground is :*

(A) 89.14 ms^{-1} *(B) 79.14 ms^{-1}* *(C) 99.14 ms^{-1}* *(D) 109 ms^{-1}*

Sol. (C)

(C) DISPLACEMENT

This displacement of the particle is expressed by

$S = x\hat{i} + y\hat{j}$ where $|S| = \sqrt{x^2 + y^2}$

$= (ut)\hat{i} + (\tfrac{1}{2}gt^2)\hat{j}$

(D) TIME OF FLIGHT

Time taken by the projectile to return to ground.

From eqn. of motion $S = ut + \tfrac{1}{2}at^2$, we get for vertical direction

$-h = v_y t - \tfrac{1}{2}gt^2$

At highest point, $v_y = 0$

$h = \tfrac{1}{2}gt^2$

$t = \pm\sqrt{\dfrac{2h}{g}}$

$= \sqrt{\dfrac{2h}{g}}$ [(-)ve time is not possible)]

Ex. *A bomb is dropped from an aeroplane flying horizontally with a velocity of 720 km/hr at an altitude of 980 m. The bomb will hit the ground after a time :*

(A) 1s *(B) 7.2 s* *(C) 14.15 s* *(D) 0.15 s*

Sol. (C)

(E) HORIZONTAL RANGE

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

$R = u_x.t$

$= u\sqrt{\dfrac{2h}{g}}$

PROJECTILE MOTION

Ex. *An aeroplane is flying horizontally at a height of 490 m with a velocity of 150 ms^{-1}. A bag containing food is to be dropped to the jawans on the ground. How far from them should the bag be dropped so that it directly reaches them ?*

(A) 1000 m *(B) 1500 m* *(C) 750 m* *(D) 2000 m*

Sol. (B)

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 490}{9.8}} = 10s$$

$\therefore$ R = 150 × 10 = 1500 m.

(F) ANGLE MADE BY FINAL VELOCITY WITH GROUND

As from fig, $\tan\beta = \frac{\vec{v}_y}{\vec{v}_x}$

CONCEPT

Projectile thrown from a moving body gains velocity of moving body only, not it's acceleration.

MISCELLANEOUS EXAMPLES BASED ON HORIZONTAL PROJECTILE MOTION

Ex.1 The maximum range of a gun on horizontal terrain is 16 km. If g = 10 m/s^2, what must be the muzzle velocity of the shell-

(A) 200 m/s (B) 400 m/s (D) 100 m/s (D) 50 m/s

Sol. (B)

$$R_{max} = \frac{u^2}{g}$$

$$= 16 \times 10^3$$

$\Rightarrow$ u = 400 m/s.

Ex.2 A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following-

(A) Straight path (B) Circular path (C) Parabolic path (D) Hyperbolic path

Sol. (C)

Due to constant velocity along horizontal and vertical downward force of gravity, stone will hit the ground following parabolic path.

Ex.3 A bullet is dropped from the same height when another bullet is fired horizontally. They will hit the ground-

(A) One after the other (B) Simultaneously

(C) Depends on the observer (D) None of the above

Sol. (B)

Because the vertical components of velocities of both the bullets are same and equal to zero and

$t = \sqrt{\frac{2h}{g}}$.

PROJECTILE MOTION

Ex.4 An aeroplane is flying at a constant horizontal velocity of 600 km/hr at an elevation of 6 km towards a point directly above the target on the earth's surface. At an appropriate time, the pilot releases a ball so that it strikes the target at the earth. The ball will appear to be falling-

(A) On a parabolic path as seen by pilot in the plane.

(B) Vertically along a straight path as seen by an observer on the ground near the target

(C) On a parabolic path as seen by an observer on the ground near the target.

(D) On a zig-zag path as seen by pilot in the plane.

Sol. (C)

The pilot will see the ball falling in straight line because the reference frame is moving with the same horizontal velocity but the observer at rest will see the ball falling in parabolic path.

Ex.5 A bomb is dropped from an aeroplane moving horizontally at constant speed. When air resistance is taken into consideration, the bomb-

(A) Falls to earth exactly below the aeroplane

(B) Falls to earth behind the aeroplane

(C) Falls to earth ahead of the aeroplane.

(D) Flies with the aeroplane.

Sol. (B)

Due to air resistance, it's horizontal velocity will decrease so it will fall behind the aeroplane.

Ex.6 A man projects a coin upwards from the gate of a uniformly moving train. The path of coin for the man will be-

(A) Parabolic (B) Inclined straight line

(C) Vertical straight line (D) Horizontal straight line

Sol. (C)

Because horizontal velocity is same for coin and the observer. So relative horizontal displacement will be zero.

PROJECTILE MOTION

PRACTICE QUESTIONS BASED ON HORIZONTAL PROJECTILE MOTION

Q.1 A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following a :

(A) Straight line path (B) Circular path (C) Parabolic path (D) Hyperbolic path

Q.2 Two bullets are fired horizontally with different velocities from the same height. Which will reach the ground first ?

(A) Slower one (B) Faster one

(C) Both will reach simultaneously (D) It cannot be predicted

Q.3 An aeroplane moving horizontally with a speed of 180 km/hr drops a food packet while flying at a height of 490 m. The horizontal range is :

(A) 180 m (B) 980 m (C) 500 m (D) 670 m

Q.4 A bomb is dropped from an aeroplane when it is directly above a target at a height of 1254.4 m. The aeroplane is going horizontally with a speed of 150 m/s. The distance by which it will miss the target is :

(A) 1.2 km (B) 2.4 km (C) 1.8 km (D) 2.8 km

Q.5 When a particle is thrown horizontally, the resultant velocity of the projectile at any time t is given by :

(A) gt (B) $\frac{1}{2}gt^2$ (C) $\sqrt{u^2+g^2t^2}$ (D) $\sqrt{u^2-g^2t^2}$

Q.6 Two tall buildings are 30 m apart. The speed with which a ball must be thrown horizontally from a window 150 m above the ground in one building so that it enters a window 27.5 m from the ground in the other building is :

(A) 2 ms^{-1} (B) 6 ms^{-1} (C) 4 ms^{-1} (D) 8 ms^{-1}

Q.7 Two paper screens A and B are separated by 150 m. A bullet pierces A and then B. The hole in B is 15 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is : (g = 10 ms^{-2})

(A) $100\sqrt{3}$ ms^{-1} (B) $200\sqrt{3}$ ms^{-1} (C) $300\sqrt{3}$ ms^{-1} (D) $500\sqrt{3}$ ms^{-1}

ANSWERS

1. (C) **2.** (C) **3.** (C) **4.** (B) **5.** (C) **6.** (B)
7. (D)

MORE EXAMPLES BASED ON HORIZONTAL PROJECTILE MOTION

Ex.1 An aeroplane is flying in a horizontal direction with a velocity of 600 km/hour and at a height of 1960 m. When it is vertically below the point A, on the ground, a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB.

Sol. The velocity of plane in horizontal direction,

v_x = 600 km/hour

$= 600 \times \frac{5}{18}$ m/s $= \frac{500}{3}$ m/s

Due to inertia, this is also the velocity of body which remains constant during the flight of body.
Initial velocity of body in vertical direction, $u_y = 0$.
If t is time taken by the body to reach the earth, then from relation

$s = ut + \frac{1}{2}at^2$, we have

$h = \frac{1}{2}gt^2$ or $t = \sqrt{\left(\frac{2h}{g}\right)}$

$= \sqrt{\left(\frac{2 \times 1960}{9.8}\right)}$ = 20 sec.

∴ Distance traversed by body in horizontal direction,

$AB = v_x t = \frac{500}{3} \times 20 = \frac{10}{3} \times 10^3$ m = 3.333 km.

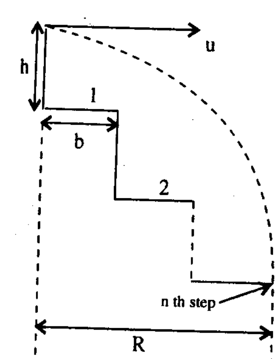
Ex.2 A ball rolls off the top of a stairway with a horizontal velocity u m/s. If the steps are h meters high and b meters wide, show that the ball will just hit the edge of nth step if

$n = \frac{2hu^2}{gb^2}$.

Sol. If the ball hits the nth step, the horizontal and vertical distances traversed are nb and nh respectively.
Let t be the time taken by the ball for these horizontal and vertical displacements. Then, velocity along horizontal direction remains constant = u, while initial vertical velocity is zero.

∴ nb = ut .... (1)

$nh = 0 + \frac{1}{2}gt^2$ .... (2)



From (1), t = nb/u
∴ Substituting this value in (2), we get

$nh = \frac{1}{2}g\left(\frac{nb}{u}\right)^2$

This gives,

$n = \frac{2hu^2}{gb^2}$.

Ex.3 A body is thrown horizontally from the top of a tower and strikes the ground after three seconds at an angle of 45º with the horizontal. Find the height of the tower and the speed with which the body was projected. Take g = 9.8 m/s².

Sol. As shown in the figure of example, $v_y = 0$ and $a_y = g = 9.8$ m/s²

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

$$= 0 \times 3 + \frac{1}{2} \times 9.8 \times (3)^2$$

$$= 44.1 \text{ m}$$

Further, $v_y = u_y + a_y t = 0 + (9.8)(3)$
$= 29.4$ m/s

As the resultant velocity v makes an angle of 45º with the horizontal, so

$$\tan 45º = \frac{v_y}{v_x} \text{ or } 1 = \frac{29.4}{v_x}$$

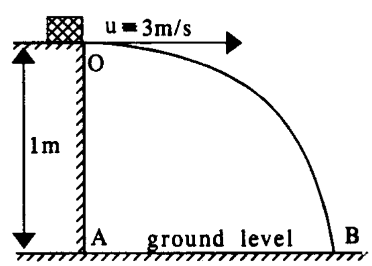
$v_x = 29.4$ m/s

Therefore, the speed with which the body was projected (horizontally) is 29.4 m/s.

Ex.4 A block slides off a horizontal table top, 1m high with a speed of 3m/s. Find :
(a) The horizontal distance from the edge of the table at which the block strikes the floor.
(b) The horizontal and vertical components of its velocity when it reaches the floor.

Sol. In the interval from O to B

$u_x = 3$ m/s
$u_y = 0$ m/s
$s_y = -1$m



$$\Rightarrow \quad t = \sqrt{\frac{2}{g}} = \frac{\sqrt{10}}{7} \text{ s.}$$

(a) $s_x = u_x t = 3 \frac{\sqrt{10}}{7}$ m = AB

= horizontal distance

(b) $v_x = u_x = 3$ m/s

$$\Rightarrow \quad v_y = u_y + a_y t = 0 - 9.8 \left(\sqrt{10}/7\right)$$

$$\Rightarrow \quad v_y = -1.4\sqrt{10} \text{ m/s}$$

horizontal component = $u_x = 3$ m/s = v_x

vertical component = $v_y = -1.4\sqrt{10}$ m/s

Ex.5 An aeroplane flying horizontally with a speed of 49 m/s releases a bomb at a height of 490 m. Find the time taken by the bomb to reach the ground and also the magnitude and the direction of velocity with which it strikes the ground.

Sol. $T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 490}{9.8}} = 10$ sec

$v_x = u_x = 49$ m/s
$v_y = u_y + gt = 0 + 9.8 \times 10 = 98$ m/s

$\therefore$ v = $\sqrt{v_x^2 + v_y^2}$ = $\sqrt{(49)^2 + (98)^2}$

= 49$\sqrt{5}$ = 109.6 m $\Rightarrow$ tan θ = $\frac{v_y}{v_x}$ = $\frac{98}{49}$ = 2

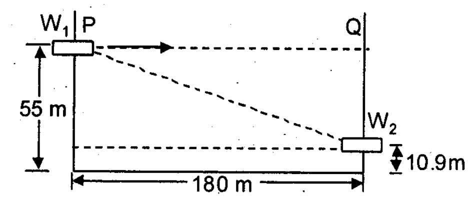
$\Rightarrow$ θ = $\tan^{-1}$ (2) below horizontal.

Ex.6 Two tall buildings face each other and are at a distance of 180m from each other. With what velocity, must a ball be thrown horizontally from a window, 55m above the ground in one building, so that it enters a window 10.9m above the ground in the second building.

Sol. In figure, P and Q are two tall buildings which are 180m apart. W_1 and W_2 are the two windows in P and Q respectively. Vertically downward distance to be covered by the ball

= Height of W_1 - Height of W_2

= 55 - 10.9 = 44.1 m



Initial vertical velocity of ball, u_y = 0

As y = $u_y t + \frac{1}{2}gt^2$

$\therefore$ 44.1 = 0 + ½ x 9.8t^2 or t^2 = $\frac{44.1 \times 2}{9.8}$ = 9

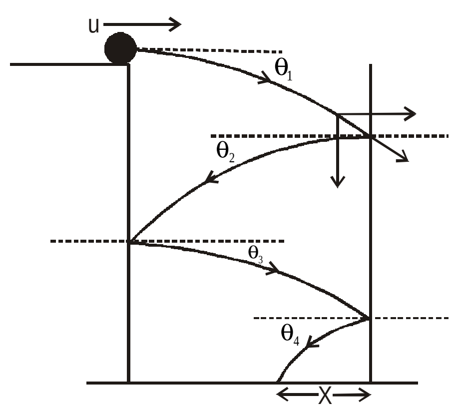
or t = 3 sec

Required horizontal velocity = $\frac{\text{Horizontal distance}}{\text{Time}}$ = $\frac{180\,\text{m}}{3\,\text{s}}$ = 60 ms^{-1}.

Ex.7 As shown in the figure, a ball falls in the well and rebounds from the wall many times. Given that u = 10 m/sec, H = 500 m, d = diameter of the well = 7 m. Calculate :

(a) No of collisions made by the ball with the wall.

(b) Range of the last rebounded path i.e. x.



Sol. $\frac{1}{2}gt^2$ = 500 $\therefore$ t = 10 sec

distance travelled by ball = velocity of ball × time taken = 10 x 10 = 100 m

$\therefore$ no. of collisions = Distance travelled by ball / Diameter of the well = $\frac{100}{7}$ = 14 (approx.)

remainder = 2 m

$\therefore$ x = 2 m

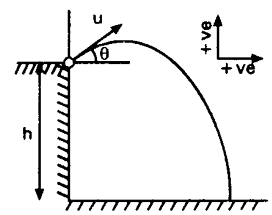
CASE (II)

PROJECTION AT AN ANGLE θ ABOVE HORIZONTAL

$u_x = u \cos \theta$

$u_y = u \sin \theta$

$a_y = - g$



Eqn. of horizontal motion :

$x = u \cos \theta \ t$ .... (1)

Eqn. of vertical motion :

$- h = u \sin \theta \ t - \frac{1}{2} gt^2$ .... (2)

From Eqn. (1) and (2),

$gt^2 - 2u \sin\theta t - 2h = 0$

or $t = \frac{u \sin\theta}{g} \pm \sqrt{\frac{u^2 \sin^2 \theta}{g^2} + \frac{2h}{g}}$

Ex.1 *A ball is projected upwards from the top of a tower with a velocity of 50 ms^{-1} making an angle of 30º with the horizontal. The height of the tower is 70m. After how many seconds from the instant of throwing will the ball reach the ground ?*

(A) 2s *(B) 5s* *(C) 7s* *(D) 9s*

Sol. (C)

The vertically upward component of the velocity of projection = 50 sin 30º m/s = 25 ms^{-1}

If t is the time taken to reach the ground, we have

$s = u_0 t + \frac{1}{2} gt^2$

or $70 = - 25 \times t + \frac{1}{2} \times 10 \times t^2$

or $5t^2 - 25t - 70 = 0$ so, t = – 2s or t = 7s

Here, t = – 2s is not valid. so, t = 7 sec

Ex.2 *A particle is projected under gravity with velocity $\sqrt{2ag}$ from a point at a height h above the level plane at an angle θ to it. The maximum range R on the ground is :*

(A) $\sqrt{(a^2 + 1)h}$ *(B)* $\sqrt{a^2 h}$ *(C)* $\sqrt{ah}$ *(D)* $2\sqrt{a(a+h)}$

Sol. (D)

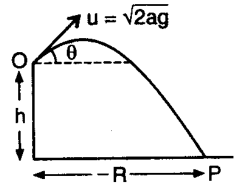
Coordinates of point P are (R, – h)

Hence $- h = R \tan \theta - \frac{gR^2}{2(2ga)} (1 + \tan^2 \theta)$

or $R^2 \tan^2\theta - 4aR\tan\theta + (R^2 - 4ah) = 0$

For θ to be real,

$(4aR)^2 \geq 4R^2(R^2 - 4ah)$



or $4a^2 \geq (R^2 - 4ah)$

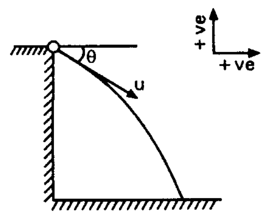
or $R^2 \leq 4a(a + h)$

or $R \leq 2\sqrt{a(a+h)}$

$\therefore$ $R_{max.} = 2\sqrt{a(a+h)}$

<u>CASE (III)</u>

PROJECTION AT AN ANGLE θ BELOW HORIZONTAL



$u_x = u\cos\theta$

$u_y = -u\sin\theta$

$a_y = -g$

Similarly, for projection at an angle θ downwards with horizontal, the eqns. are

Eqn. of horizontal motion :

$$x = u\cos\theta\, t \qquad ....(1)$$

Eqn. of vertical motion :

$$-h = -u\sin\theta\, t - \frac{1}{2}gt^2 \qquad ....(2)$$

from equation (2),

or $gt^2 + 2u\sin\theta\, t - 2h = 0$

or $t = \dfrac{-2u\sin\theta \pm \sqrt{4u^2\sin^2\theta + 8gh}}{2g}$ Neglect -ve root of t, as negative value of t has no meaning.

NOTE : In all the above three cases, we can calculate the velocity of projectile at the instant of striking the ground by using

$$V = \sqrt{V_x^2 + V_y^2}$$

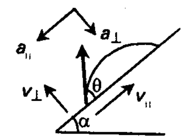
$\tan\theta = \dfrac{V_y}{V_x}$ [angle at which projectile strikes ground]

PROJECTILE ON AN INCLINED PLANE

Now, we are considering the motion of a projectile on an inclined plane which makes an angle α with the horizontal. Projectile makes angle θ with the inclined plane.

The various parameters can be represented as :

(1) TIME OF FLIGHT



Here $v_\perp = v_o \sin\theta$

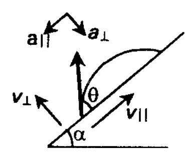
and $a_\perp = g \cos\alpha$

Thus, $T = \dfrac{2v_\perp}{a_\perp} = \dfrac{2v_o \sin\theta}{g\cos\alpha}$

(2) RANGE ALONG THE INCLINED PLANE

The range of the projectile along the inclined plane is given by

$$R' = v_{||}T - \frac{1}{2}a_{||}T^2$$



Since $T = \dfrac{2v_\perp}{a_\perp} = \dfrac{2v_o \sin\theta}{g\cos\alpha}$

On solving, we get :

$\therefore$ $R' = \dfrac{2v_o^2}{g}\dfrac{\sin\theta\cos(\theta+\alpha)}{\cos^2\alpha}$ [Putting $v_{11} = v_0 \cos\theta$ and $a_{11} = g \sin\alpha$]

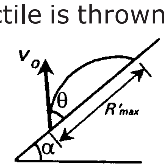
IMPORTANT POINTS

(a) The maximum range occurs when

$$\theta = \frac{\pi}{4} - \frac{\alpha}{2}$$

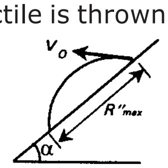
(b) The maximum range along the inclined plane when the projectile is thrown upwards is given by

$$R'_{max} = \frac{v_o^2}{g(1+\sin\alpha)}$$



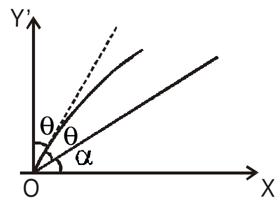
(c) The maximum range along the inclined plane when the projectile is thrown downwards is given by

$$R''_{max} = \frac{v_o^2}{g(1-\sin\alpha)}$$



(d) For maximum range in inclined projectile, the direction of projection bisects the angle that the inclined plane makes with vertical direction to ground (OY')

i.e. $\frac{\pi}{2} - (\theta + \alpha) = \theta$.



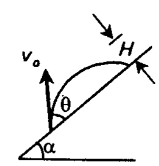
(3) MAXIMUM HEIGHT OF THE PROJECTILE

If a projectile is thrown up an inclined plane, as shown in the **fig.** maximum height attained is given by

$$H = \frac{v_{\perp}^2}{2a_{\perp}} \quad \Rightarrow \quad H = \frac{(v_o \sin\theta)^2}{2g\cos\alpha} = \frac{v_o^2 \sin^2\theta}{2g\cos\alpha}$$

For projectile up the inclined plane, we can use these formulae which are applicable to 'same-level-horizontal-projectile', also

$$T = \frac{2V_{\perp}}{a_{\perp}}; \; R' = V_{11}\,T - \frac{1}{2}\,a_{11}\,T^2; \; H = \frac{V_{\perp}^2}{2a_{\perp}}$$



maximum range occurs at $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$ i.e. (θ < 45º)

$$R'_{max}\text{ (up the plane)} = \frac{v_0^2}{g(1+\sin\alpha)},\; R'_{max}\text{ (down the plane)} = \frac{v_0^2}{g(1-\sin\alpha)};\; H = \frac{v_0^2\sin^2\theta}{2g\cos\alpha}$$

NOTE : The angle of projectile θ is measured from inclined plane, not ground.

Ex.1 *A particle is projected with a certain velocity at an angle α above the horizontal from the foot of an inclined plane of inclination 30º. If the particle strikes the plane normally then α is equal to :*

(A) 30º + $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ *(B) 45º* *(C) 60º* *(D) 30º + $\tan^{-1}(2\sqrt{3})$*

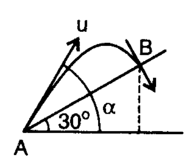
Sol. (A)

t_{AB} = time of flight of projectile = $\frac{2u\sin(\alpha - 30º)}{g\cos 30º}$

Now component of velocity along the plane becomes zero at point B.

$\therefore$ 0 = u cos (α – 30º) – g sin 30º × T

or u cos (α – 30º)



$= g \sin 30º \times \frac{2u\sin(\alpha - 30º)}{g\cos 30º}$

or $\tan(\alpha - 30^\circ) = \dfrac{\cot 30^\circ}{2} = \dfrac{\sqrt{3}}{2}$

$\therefore \quad \alpha = 30^\circ + \tan^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$

Ex.2 *A particle is thrown on a plane inclined at an angle of 30º such that the projectile makes angle of 60º with ground. Time taken by the projectile to reach from A to B on inclined plane is t. Then the distance AB is equal to-*

(A) $\dfrac{ut}{\sqrt{3}}$ *(B)* $\dfrac{\sqrt{3}\,ut}{2}$ *(C)* $\sqrt{3}\,ut$ *(D) 2ut*

Sol. (A)

Horizontal component of velocity,

$u_H = u \cos 60^\circ = \dfrac{u}{2}$

$\therefore \quad AC = u_H \times t = \dfrac{ut}{2}$ (where C is the point on ground vertically below point B on inclined plane)

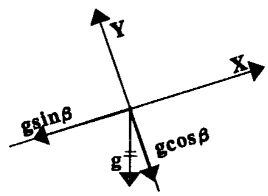
and $AB = AC \sec 30^\circ$

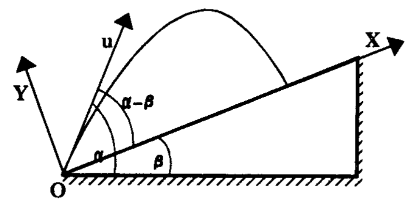
$= \left(\dfrac{ut}{2}\right)\left(\dfrac{2}{\sqrt{3}}\right) = \dfrac{ut}{\sqrt{3}}$.

MORE EXAMPLES BASED ON PROJECTION ON AN INCLINED PLANE

Ex.1 A particle is thrown with a velocity u at an angle α with the horizontal from the bottom of an inclined plane. Taking X-axis parallel to the plane and the Y-axis perpendicular to the plane, find the range and the time of flight on the inclined plane. The inclined plane makes an angle β with the horizontal.

Sol. Taking axes along and perpendicular to the inclined plane as shown, components of g are g cosβ and g sinβ as shown.





From O to A :

$s_y = 0 \quad \Rightarrow \quad 0 = u_y t + 1/2\, a_y t^2$

$\Rightarrow \; 0 = u \sin(\alpha - \beta)\, t - \frac{1}{2} g \cos\beta\, t^2$

$\Rightarrow \; t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$ is the time of flight.

$\Rightarrow \; a_x = -g\sin\beta; \qquad a_y = -g\cos\beta$

$u_x = u\cos(\alpha - \beta); \quad u_y = u\sin(\alpha - \beta)$

Range = OA $\Rightarrow s_x = u_x t + \frac{1}{2} a_x t^2$

substituting values of u_x, a_x and t, we get

$$\text{Range} = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$$

Ex.2 A particle is projected with a velocity 39.2 m/sec at an angle of 30º to an inclined plane (inclined at an angle of 45º to the horizontal). Find the range on the incline (a) when it is projected upward (b) when it is projected downward.

Sol. Time of flight will be same in both cases, because the acceleration perpendicular to the plane is same. Therefore

$$0 = 39.2 \sin 30^\circ\, t - (½) g \cos 45^\circ\, t^2$$

or $\quad t = \frac{2 \times 39.2 \sin 30^\circ}{g\cos 45^\circ} = 4\sqrt{2}$ sec

(a) Range upward

$= 39.2 \cos 30^\circ\, t - (½)\, g \sin 45^\circ\, t^2$

$= 39.2 \times \frac{\sqrt{3}}{2} \times 4\sqrt{2} - \frac{1}{2} \times 9.8 \times \frac{1}{\sqrt{2}} \times (4\sqrt{2})^2 = 80.026$ m.

(b) Range downward $= 39.2 \cos 30^\circ \times t + (½)\, g \sin 45^\circ\, t^2$

$= 39.2 \times \frac{\sqrt{3}}{2} \times 4\sqrt{2} + \frac{1}{2} \times 9.8 \times \frac{1}{\sqrt{2}} \times (4\sqrt{2})^2 = 301.883$ m.

Ex.3 Two bodies are projected from the same point with equal speeds in such directions that they both strike the same point on a plane whose inclination is β. If α be the angle of projection of the first body with the horizontal, show that the ratio of their times of flight is

$$\frac{\sin(\alpha-\beta)}{\cos\alpha}$$

Sol. Let α' be the angle of projection of the second body

$$R = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha-\beta)-\sin\beta]$$

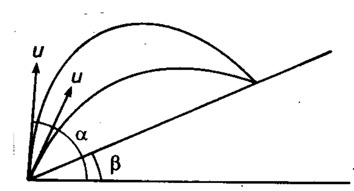
Range of both the bodies is same. Therefore,

$$\sin(2\alpha-\beta) = \sin(2\alpha'-\beta)$$

or $2\alpha'-\beta = \pi-(2\alpha-\beta)$

$$\alpha' = \frac{\pi}{2}-(\alpha-\beta)$$

Now, $$T = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$



and $$T' = \frac{2u\sin(\alpha'-\beta)}{g\cos\beta}$$

$$\therefore \quad \frac{T}{T'} = \frac{\sin(\alpha-\beta)}{\sin(\alpha'-\beta)} = \frac{\sin(\alpha-\beta)}{\sin\left\{\frac{\pi}{2}-(\alpha-\beta)-\beta\right\}}$$

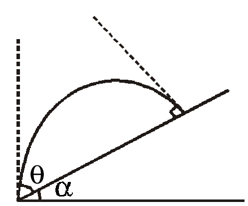
$$= \frac{\sin(\alpha-\beta)}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \frac{\sin(\alpha-\beta)}{\cos\alpha}$$

EX.4 A projectile is projected up the inclined plane of inclination α, making an angle θ from inclined plane. It strikes the inclined plane perpendicular to inclined surface. Find the value of θ in terms of α.

Sol. Horizontal component at the time of striking becomes zero, so

$$0 = u\cos\theta - g\sin\alpha \cdot \frac{2u\sin\theta}{g\cos\alpha}$$

$$\therefore \tan\theta = \frac{\cot\alpha}{2}$$



Ex.5 With what minimum speed must a particle be projected from origin, so that it is able to pass through a given point P (a, b) ?

Sol. Let u and θ be the velocity and angle of projection respectively.

For the projectile to pass through P(a, b).

$$b = a \tan\theta - \frac{ga^2}{2u^2}(1 + \tan^2\theta)$$

or $ga^2 \tan^2\theta - 2au^2 \tan\theta + (ga^2 + 2bu^2) = 0$

The projectile will pass through P(a, b), if this equation (quadratic in $\tan\theta$) gives some real value of θ, i.e. its discriminant ≥ 0.

$4a^2 u^4 - 4ga^2 (ga^2 + 2bu^2) \geq 0$

$u^4 - 2gbu^2 - g^2 a^2 \geq 0$

$u^4 - 2gbu^2 + b^2 g^2 \geq b^2 g^2 + a^2 g^2$

$(u^2 - bg)^2 \geq (b^2 + a^2) g^2$

$\Rightarrow$ $u \geq \sqrt{bg + g\sqrt{a^2 + b^2}}$

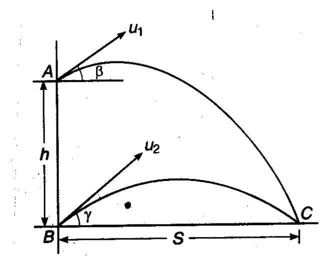
i.e., the minimum value of u is $\sqrt{bg + g\sqrt{a^2 + b^2}}$

Ex.6 Two shots are fired simultaneously from the top and bottom of a vertical tower AB at angles β and γ with horizontal respectively. Both shots strike at the same point C on the ground at distance 'S' from the foot of the tower at the same time. Show that the height of the tower is S(tan γ – tan β).

Sol. Let, height of tower be h

$$-h = u_1 \sin\beta \cdot t - \frac{1}{2}gt^2$$

$$0 = u_2 \sin\gamma \cdot t - \frac{1}{2}gt^2$$



or $u_1 \sin\beta \cdot t + h = u_2 \sin\gamma \cdot t$

and $S = u_1 \cos\beta\, t = u_2 \cos\gamma\, t$

$\therefore$ $t = \dfrac{S}{u_1 \cos\beta} = \dfrac{S}{u_2 \cos\gamma}$

$\therefore$ $u_1 \sin\beta \cdot \dfrac{S}{u_1 \cos\beta} + h = \dfrac{u_2 \sin\gamma}{u_2 \cos\gamma} \cdot S$

$h + S \tan\beta = S \tan\gamma$

$h = S (\tan\gamma - \tan\beta)$

Ex.7 A body falling freely from a given height H, hits an inclined plane in its path at a height 'h' . As a result of this impact, the direction of the velocity of the body becomes horizontal. For what value of (h/H), the body will take maximum time to reach the ground ?

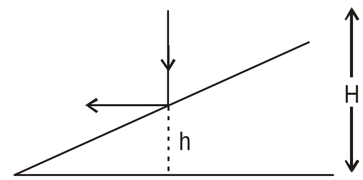
Sol. In accordance with Fig. time taken by the body to strike the inclined plane

$$t_1 = \sqrt{\frac{2(H-h)}{g}}$$

Now as after impact, the velocity of the body is horizontal, so time taken to reach the ground

$$t_2 = \sqrt{\frac{2h}{g}}$$

So total time of motion



$$t = t_1 + t_2 = \sqrt{\frac{2}{g}}\left[\sqrt{h} + \sqrt{(H-h)}\right]$$

For t to be maximum (dt/dh) = 0

i.e., $$\frac{d}{dh}\left[\frac{\sqrt{2}}{g}\{h^{1/2} + (H-h)^{1/2}\}\right] = 0$$

or $$\frac{1}{2}h^{-1/2} + \frac{1}{2}(H-h)^{-1/2}(-1) = 0 \qquad \left[\text{as} \frac{\sqrt{2}}{g} \neq 0\right]$$

or $h = H - h$, i.e., $\frac{h}{H} = \frac{1}{2}$

Ex.8 A particle is projected from point O on the ground with velocity $u = 5\sqrt{5}$ m/s at angle $\alpha = \tan^{-1}$ (0.5). It strikes at a point C on a fixed smooth plane AB having inclination of 37º with horizontal as shown in fig. If the particle does not rebound, calculate

(a) Coordinates of point C in reference to coordinate system as shown in the figure.

(b) Maximum height from the ground to which the particle rises. ($g = 10$ m/s^2).

Sol. (a) Let (x, y) be the coordinates of point C.

x = OD = OA + AD.

$$\therefore \quad x = \frac{10}{3} + y \cot 37^\circ = \frac{10+4y}{3} \qquad \text{........... (i)}$$

As point C lies on the trajectory of a parabola, we have

$$y = x \tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha) \qquad \text{........... (ii)}$$

Given that, $\tan\alpha = 0.5$.

Solving eqs. (i) and (ii), we get x = 5m and y = 1.25 m.

Hence, the coordinates of point C are (5m, 1.25 m)

(b) Let v_y be the vertical component of velocity of the particle just before collision at C.

Using v = u + at, we have

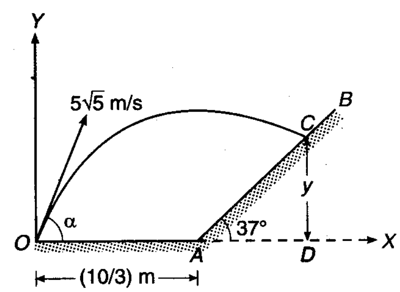
$v_y = u \sin \alpha - g (x/u \cos \alpha)$ $\quad (\because \quad t = x/u \cos \alpha)$

$= \frac{5\sqrt{5}}{\sqrt{5}} - \frac{10 \times 5}{\left(5\sqrt{5} \times 2/\sqrt{5}\right)} = 0$

Thus, at C, the particle has only horizontal component of velocity

$v_x = u \cos \alpha = 5\sqrt{5} \times (2/\sqrt{5}) = 10$ m/s

Given, that the particle does not rebound after collision. So, the normal component of velocity (normal to the plane AB) becomes zero. Now, the particle slides up the plane due to tangential component $v_x \cos 37^o = (10)\left(\frac{4}{5}\right) = 8$ m/s.



Let h be the further height raised by the particle. Then,

$mgh = \frac{1}{2} m(8)^2$ or h = 3.2 m

Height of the particle from the ground = y + h

H = 1.25 + 3.2 = 4.45 m

Ex.9 Two parallel straight lines are inclined to the horizontal at an angle α. A particle is projected from a point mid way between them so as to graze one of the lines and strikes the other at right angles. Show that if θ is the angle between the direction of projection and either of lines, then

$\tan \theta = (\sqrt{2} - 1) \cot \alpha$

Sol. Consider the motion of the particle from O to P.

The velocity v_y at P is zero.

$v_y^2 = u_y^2 + 2a_y s_y$

$\therefore \quad 0 = (u \sin \theta)^2 - 2 (g \cos \alpha)\, b$

or $b = \frac{u^2 \sin^2 \theta}{2g \cos \alpha}$ .... (i)

Now, consider the motion of the particle from O to Q.

The particle strikes the point Q at 90º to AB i.e. its velocity along x-direction is zero.

Using $v_x = u_x + a_x t$, we have

$0 = u \cos \theta - (g \sin \alpha)\, t$ .... (ii)

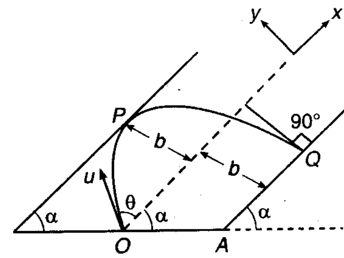
or $t = \frac{u \cos \theta}{g \sin \alpha}$

From motion in y-direction,

or $$-b = u\sin\theta\left(\frac{u\cos\theta}{g\sin\alpha}\right) + \frac{1}{2}(-g\cos\alpha)\left(\frac{u\cos\theta}{g\sin\alpha}\right)^2 \quad \text{.........(iii)}$$

From Eqs. (i) and (iii)

or $$-\frac{u^2\sin^2\theta}{2g\cos\alpha} = \frac{u^2\sin\theta\cos\theta}{g\sin\alpha} - \frac{gu^2\cos\alpha\cos^2\theta}{2g^2\sin^2\alpha}$$



or $$\frac{\sin^2\theta}{2\cos\alpha} = \frac{\sin\theta\cos\theta}{\sin\alpha} - \frac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$$

Solving, we get $\tan\theta = (\sqrt{2} - 1)\cot\alpha$

Ex.10 Two inclined planes OA and OB having inclinations 30º and 60º with the horizontal respectively intersect each other at O, as shown in figure. A particle is projected from point P with velocity u $= 10\sqrt{3}\ \frac{m}{s}$ along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicularly at Q. Calculate.

(a) Time of flight

(b) Velocity with which the particle strikes the plane OB.

(c) Height h of point P from point O.

(d) Distance PQ. (Take $g = 10\ m/s^2$).

Sol. Let us choose the x and y directions along OB and OA respectively. Then

$u_x = u = 10\sqrt{3}$ m/s, $u_y = 0$

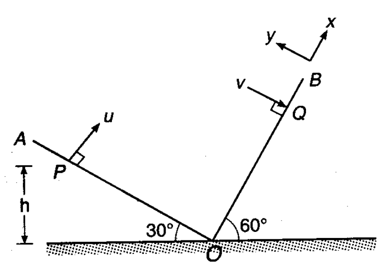
$a_x = -g\sin 60º = -5\sqrt{3}\ m/s^2$

and $a_y = -g\cos 60º = -5\ m/s^2$

(a) At point Q, x-component of velocity is zero. Hence, substituting in

$v_x = u_x + a_x t$

$0 = 10\sqrt{3} - 5\sqrt{3}\,t.$



or $t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2s$

(b) At point Q, $v = v_y = u_y + a_y t$

∴ $v = 0 - (5)(2) = -10$ m/s

Here, negative sign implies that velocity of particle at Q is along negative y direction.

(c) Distance PO = | displacement of particle along y-direction | = | s_y |

Here, $s_y = u_y t + \frac{1}{2} a_y t^2$

$= 0 - \frac{1}{2}(5)(2)^2 = -10$ m

$\therefore$ PO = 10 m

Therefore, $h = PO \sin 30^\circ = (10)\left(\frac{1}{2}\right)$

or $h = 5m$

(d) Distance OQ = displacement of particle along x-direction = s_x

Here, $s_x = u_x t + \frac{1}{2} a_x t^2$

$= (10\sqrt{3})(2) - \frac{1}{2}(5\sqrt{3})(2)^2$

$= 10\sqrt{3}$ m

or $OQ = 10\sqrt{3}$ m

$\therefore$ $PQ = \sqrt{(PO)^2 + (OQ)^2} = \sqrt{(10)^2 + (10\sqrt{3})^2}$

$= \sqrt{100 + 300} = \sqrt{400}$

$\therefore$ PQ = 20 m.

PROJECTILE WITH VARIABLE ACCELERATION

Suppose a projectile moves in the two dimensional plane with velocity $v = a\hat{i} + bx\hat{j}$ where a and b are constant. Initially, consider the particle to be situated at origin i.e. at x = 0 & y = 0. Now, let us first find out the equation of trajectory of the projectile. So.

$$v = a\hat{i} + bx\hat{j}$$

$\therefore$ $v_x = a$ and $v_y = bx$

But $v = \dfrac{dr}{dt}$

i.e. $\dfrac{dx}{dt} = a$ and $\dfrac{dy}{dt} = bx$

$x = at$ and $dy = bxdt$

On substituting value of x we have

$$dy = b.\ at\ dt$$

On integrating

$$y = \frac{abt^2}{2}$$

$$y = \frac{ab}{2}\left(\frac{x}{a}\right)^2$$

$$y = \frac{bx^2}{2a}$$

This is the equation of trajectory of the projectile.

Now, radius of curvature of trajectory is given by,

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Since $y = \dfrac{b}{2a}.x^2$; $\dfrac{dy}{dx} = \dfrac{b}{2a}.2x = \dfrac{bx}{a}$

$$\frac{d^2y}{dx^2} = \frac{b}{a}$$

$\therefore$ $$R = \frac{\left[1+\left(\frac{bx}{a}\right)^2\right]^{3/2}}{\frac{b}{a}} = \frac{a}{b}\left[1+\left(\frac{bx}{a}\right)^2\right]^{3/2}$$

Ex. *Consider a balloon which rises from the surface of the earth. The ascension rate is constant and is given by v_o. Now the blowing action of wind causes the balloon to gather horizontal velocity component v_x = ay where y is height of ascent. Now we have to find the horizontal drift of the balloon as well as total, tangential and normal acceleration of the balloon.*

Sol. In the problem, ascension rate v_o is constant i.e.

$$\frac{dy}{dt} = v_o \qquad ...(1)$$

$$dy = v_o dt$$

$$y = v_o t$$

Now $v_x = ay$ and $v_x = \frac{dx}{dt}$

$\therefore$ $\frac{dx}{dt} = ay$

$$dx = aydt = av_o\, t\, dt$$

On integrating

$$x = \frac{av_o t^2}{2} = \frac{av_o}{2}\left(\frac{y}{v_o}\right)^2$$

$$= \frac{ay^2}{2v_o}$$

For finding out total acceleration, we have

$v_x = ay$ and $v_y = v_o$

$\therefore$ $v = \sqrt{v_x^2 + v_y^2} = \sqrt{a^2y^2 + v_o^2}$

Total tangential acceleration

$$\alpha_t = \frac{dv}{dt} = \frac{d}{dt}\sqrt{a^2y^2 + v_o^2}$$

$$= \frac{a^2y}{\sqrt{a^2y^2 + v_o^2}}.v_o \qquad \{from(1)\}$$

$$= \frac{a^2y}{\sqrt{1+(ay/v_o)^2}}$$

Now, v_y = constant

so, $\frac{dv_y}{dt} = 0$ or $\alpha_y = 0$

and $\alpha_x = \frac{dv_x}{dt} = \frac{d}{dt}(ay) = \frac{ady}{dt} = av_o$

$\therefore$ $\alpha = \sqrt{\alpha_x^2 + \alpha_y^2} = \sqrt{(av_o)^2 + 0^2}$

$\alpha = av_o$

$\therefore$ Normal acceleration :

$$\alpha_n = \sqrt{\alpha^2 - \alpha_t^2} = \sqrt{a^2 v_o^2 - \frac{(a^2 y)^2}{1 + (ay/v_o)^2}}$$

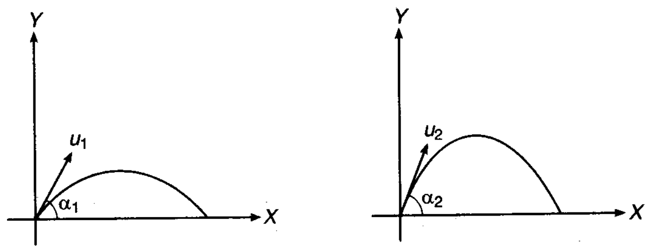
On calculating,

$$\alpha_n = -\frac{av_o}{\sqrt{1 + (ay/v_o)^2}}$$ (–ve sign indicates downward direction)

RELATIVE MOTION BETWEEN TWO PROJECTILES

Let us now discuss the relative motion between two projectiles or the path of one projectile observed by the other. Suppose that two particles are projected from the ground with speed u_1 and u_2 at angles α_1 and α_2 as shown in fig. Acceleration of both the particles is g downwards. So, relative acceleration between them is zero because

$$a_{12} = a_1 - a_2 = g - g = \text{zero.}$$



i.e., the relative motion between the two particles is uniform. Now,

$u_{1x} = u_1 \cos \alpha_1$, $u_{2x} = u_2 \cos \alpha_2$

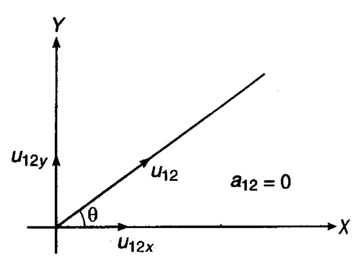
$u_{1y} = u_1 \sin \alpha_1$ and $u_{2y} = u_2 \sin \alpha_2$

Therefore, $u_{12x} = u_{1x} - u_{2x} = u_1 \cos \alpha_1 - u_2 \cos \alpha_2$

and $u_{12y} = u_{1y} - u_{2y} = u_1 \sin \alpha_1 - u_2 \sin \alpha_2$

u_{12x} and u_{12y} are the x and y components of relative velocity of 1 with respect to 2.

Hence, relative motion of 1 with respect to 2 is a straight line at an angle $\theta = \tan^{-1}\left(\frac{u_{12y}}{u_{12x}}\right)$ with positive x-axis.



Now, if $u_{12x} = 0$ or $u_1 \cos \alpha_1 = u_2 \cos \alpha_2$, the relative motion is along y-axis or in vertical direction (as $\theta = 90^\circ$). Similarly, if $u_{12y} = 0$ or $u_1 \sin \alpha_1 = u_2 \sin \alpha_2$, the relative motion is along x-axis or in horizontal direction (as $\theta = 0^\circ$).

Ex. *The path of one projectile as seen by an observer on another projectile is a/an :*

(A) Straight line *(B) Parabola* *(C) Ellipse* *(D) Circle*

Sol. (A)

We have that

$x = (u \cos \theta)t$ and $y = (u \sin \theta)\, t - \frac{1}{2}gt^2$

Let $x_2 - x_1 = (u_1 \cos \theta_1 - u_2 \cos \theta_2)t = X$

$y_2 - y_1 = (u_1 \sin \theta_1)t - \frac{1}{2}gt^2 - (u_2 \sin \theta_2)t + \frac{1}{2}gt^2$

$= (u_1 \sin \theta_1 - u_2 \sin \theta_2)t = Y$

$\therefore \quad \frac{Y}{X} = \frac{(u_1 \sin\theta_1 - u_2 \sin\theta_2)t}{(u_1 \cos\theta_1 - u_2 \cos\theta_2)t}$ = constant, m (say)

$= \frac{u_1 \sin\theta_1 - u_2 \sin\theta_2}{u_1 \cos\theta_1 - u_2 \cos\theta_2}$

$\therefore \quad Y = mX$

it is the equation of a straight line passing through the origin.

CONDITION OF COLLISION OF TWO PROJECTILES

Now, let the particles are projected simultaneously from two different heights h_1 and h_2 with speeds u_1 and u_2 in the directions shown in fig. Then the particles collide in air if relative velocity of 1 with respect to 2($\vec{u}_{12}$) is along line AB or the relative velocity of 2 with respect to 1 ($\vec{u}_{21}$) is along the line BA. Thus,

$\vec{u}_1 = u_1 \cos \alpha_1 \hat{i} + u_1 \sin\alpha_1 \hat{j}$ $\qquad$ $\vec{u}_2 = - u_2 \cos\alpha_2 \hat{i} + u_2 \sin\alpha_2 \hat{j}$

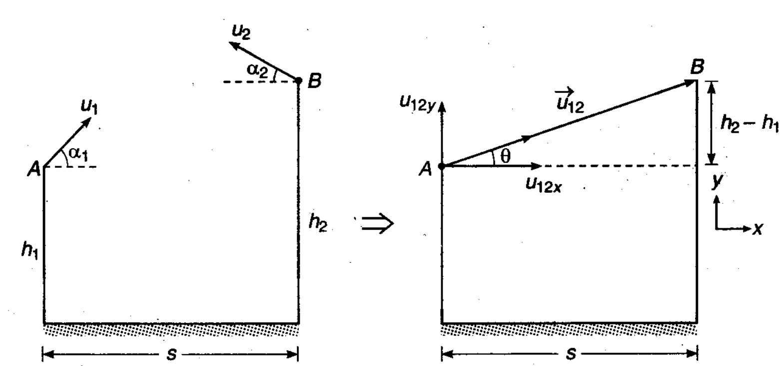
$\tan \theta = \frac{u_{12y}}{u_{12x}} = \left(\frac{h_2 - h_1}{s}\right)$

Here, $u_{12y} = u_{1y} - u_{2y} = u_1 \sin \alpha_1 - u_2 \sin \alpha_2$

and $u_{12x} = u_{1x} - u_{2x} = (u_1 \cos \alpha_1) - (- u_2 \cos \alpha_2) = u_1 \cos \alpha_1 + u_2 \cos \alpha_2$

If both the particles are initially at the same level ($h_1 = h_2$), then for collision

$u_{12y} = 0$ or $u_1 \sin \alpha_1 = u_2 \sin \alpha_2$



The time of collision of the two particles will be

$$t = \frac{AB}{|\vec{u}_{12}|} = \frac{AB}{\sqrt{(u_{12x})^2 + (u_{12y})^2}}$$

Further, the above conditions are not merely sufficient for collision to take place. For example, the time of collision discussed above should be less than the time of collision of either of the particles with the ground.

<< method of solving problems on collision >>

CONDITION OF COLLISION OR CONDITION OF MEETING TOGETHER

1. At the time of collision, coordinates of both particles should be same.
   i.e. $x_1 = x_2$ and $y_1 = y_2$ (for a 2-D motion)
   Similarly, $x_1 = x_2$, $y_1 = y_2$ and $z_1 = z_2$ (for a 3-D motion)
2. Two particles collide at the same moment. Of course, their times of journey may be different i.e. they may start at different times (t_1 and t_2 may be different). If they start together, then $t_1 = t_2$.

LEVEL # 1

Q.1 A ball is rolled off the edge of a horizontal table at a speed of 4 m/second. It hits the ground after 0.4 second. Which statement given below is true-

(A) It hits the ground at a horizontal distance of 1.6 m from the edge of the table.

(B) The speed with which it hits the ground is 4.0 m/sec.

(C) Height of the table is 0.4 m.

(D) It hits the ground at an angle of 60º to the horizontal.

Q.2 An aeroplane flying 490 m above ground level at 100 m/s, releases a block. How far on ground will it strike-

(A) 0.1 km (B) 1 km (C) 2 km (D) None

Q.3 A body is thrown horizontally from the top of a tower of height 5m. It touches the ground at a distance of 10m from the foot of the tower. The initial velocity of the body is

($g = 10ms^{-2}$).

(A) 2.5 ms^{-1} (B) 5 ms^{-1} (C) 10 ms^{-1} (D) 20 ms^{-1}

Q.4 An aeroplane moving horizontally with a speed of 720 km/h drops a food pocket, while flying at a height of 396.9 m. The time taken by a food packet to reach the ground and its horizontal range is (Take g = 9.8 m/sec^2)

(A) 3 sec and 2000 m (B) 5 sec and 500 m

(C) 8 sec and 1500 m (D) 9 sec and 1800 m

Q.5 A particle (A) is dropped from a height and another particle (B) is thrown in horizontal direction with speed of 5 m/sec from the same height. The correct statement is-

(A) Both particles will reach the ground simultaneously

(B) Both particles will reach the ground with same speed

(C) Particle (A) will reach the ground first with respect to particle (B)

(D) Particle (B) will reach the ground first with respect to particle (A)

Q.6 A particle moves in a plane with constant acceleration in a direction different from the initial velocity. The path of the particle will be-

(A) A straight line (B) An arc of a circle (C) A parabola (D) An ellipse

Q.7 At the height 80 m, an aeroplane is moving with 150 m/s. A bomb is dropped from it so as to hit a target. At what distance from the target should the bomb be dropped (given g = 10 m/s^2)

(A) 605.3 m (B) 600 m (C) 80 m (D) 230 m

Q.8 A bomber plane moves horizontally with a speed of 500 m/s and a bomb released from it strikes the ground in 10 sec. Angle at which it strikes the ground will be (g = 10 m/s^2)

(A) $\tan^{-1}\left(\frac{1}{5}\right)$ (B) $\tan^{-1}\left(\frac{1}{10}\right)$ (C) $\tan^{-1}$ (1) (D) $\tan^{-1}$ (5)

Q.9 A rifle shoots a bullet with a muzzle velocity of 400 m/sec at a small target 400 m away. The height above the target at which the bullet must be aimed to hit the target is :

(g = 10 ms^{-2})

(A) 1m (B) 5m (C) 7m (D) 10m

LEVEL # 2

MORE THAN ONE CHOICE MAY BE CORRECT :

Q.1 A ball rolls off the edge of a horizontal plane 4.9 m high. If it strikes the floor at a point 10 m horizontally away from the edge of the plane, the speed at the instant it left the plane is

(A) 10 m/s (B) 9.8 m/s (C) 4.9 m/s (D) 19.6 m/s.

Q.2 A man on the observation platform of a train moving with constant velocity drops a ball while leaning over the railing. The path of the ball as seen by an observer standing on the ground nearby is a

(A) Straight line vertically down (B) Horizontal straight line
(C) Parabola (D) Circle

Q.3 Two tall buildings are 40 m apart. With what speed, must a ball be thrown horizontally from a window 145 m above the ground in one building, so that it enters a window 22.5 m from the ground in the other ?

(A) 8 m/s (B) 16 m/s (C) 9.8 m/s (D) 19.6 m/s

Q.4 A body is thrown horizontally with velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is

(A) h (B) h/2 (C) 2h (D) 2h/3

Q.5 Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting A, then the velocity of the bullet at A is-

(A) 100 m/s (B) 200 m/s (D) 600 m/s (D) 700 m/s

Q.6 Two bullets are fired simultaneously from the same level and in the horizontal direction over a lake. The speed of one bullet is 196 m/sec. Assuming that the air friction is negligible and the lake is still, the bullet which is faster will, compared to the slower one, fall in the water-

(A) Half the time before (B) At the same time
(C) Twice the time before (D) None of the above.

Q.7 A ball is thrown from rear end of the compartment to the front end which is moving at constant horizontal velocity. An observer A sitting in the compartment and another observer B standing on the ground draw the trajectory. They will have-

(A) Equal horizontal and equal vertical ranges.
(B) Equal vertical ranges but different horizontal ranges
(C) Different vertical ranges but equal horizontal ranges
(D) Different vertical and different horizontal ranges.

Q.8 An aeroplane is in a level flight at 144 km/hr at an altitude of 1000 m. How far from a given target should a bomb be released from it to hit the target-

(A) 571.43 m (B) 671.43 m (C) 471.34 m (D) 371.34 m

Q.9 A ball rolls off the top of a stair way with a horizontal velocity 4.5 m/s. If the steps are 0.2 metre high and 0.3 metre wide, the ball will hit the edge of the nth step, where n is equal to -

(A) 6 (B) 9 (C) 10 (D) 12

Q.10 An aeroplane is flying horizontally with a velocity of 720 km/h at an altitude of 490 m. When it is just vertically above the target a bomb is dropped from it. How far horizontally it missed the target?

(A) 1000 m (B) 2000 m (C) 100 m (D) 200 m

Q.11 An aeroplane is flying in a horizontal direction with a velocity of 500 km/hr and at a height of 1500 m. When it is vertically below the point A, on the ground, a body is dropped from it. The body strikes the ground at point B. The distance AB is equal to :

(A) 2.405 km (B) 3.33 km (C) 4.5 km (D) 6 km

Q.12 An aeroplane is moving with a horizontal velocity u at a height h above the ground, if a packet is dropped from it, the speed of the packet when it reaches the ground will be-

(A) $\sqrt{u^2+2gh}$ (B) $\sqrt{2gh}$ (C) $\sqrt{u^2-2gh}$ (D) 2gh

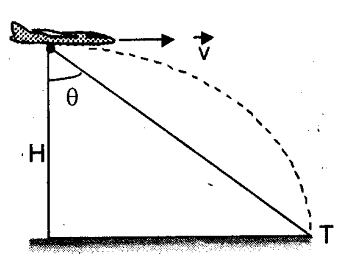
Q.13 From the top of a tower of height h, a body of mass m is projected in the horizontal direction with a velocity v. It falls on the ground at a distance x from the tower if a body of mass 2m is projected from the top of another tower of height 2h in the horizontal direction so that it falls on the ground at a distance 2x from the tower, the horizontal velocity of the second body is-

(A) 2v (B) $\sqrt{2}$ v (C) $\frac{v}{2}$ (D) $\frac{v}{\sqrt{2}}$

Q.14 A stone is thrown from a bridge at an angle of 30° down with the horizontal with a velocity of 25 m/s. If the stone strikes the water after 2.5 sec, then calculate the height of the bridge from the water surface -

(A) 61.9 m (B) 35 m (C) 70 m (D) None

Q.15 A bomber is moving with a velocity v (m/s) above H metre from the ground. The bomber releases a bomb to hit a target T when the sighting angle is θ.Then the relation between θ, H and v is -



(A) θ = $\tan^{-1} v\sqrt{2Hg}$ (B) θ = $\tan^{-1} v\sqrt{2/gH}$

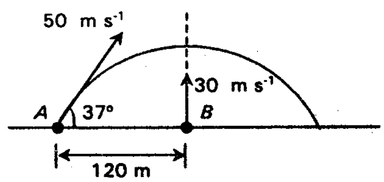
(C) θ = $\tan^{-1} v\sqrt{H/2g}$ (D) None of the above

Q.16 From the top of a tower 19.6 m high, a ball is thrown horizontally. If the line joining the point of projection to the point where it hits the ground makes an angle of 45º with the horizontal, then the initial velocity of the ball is-
(A) 9.8 ms^{-1} (B) 4.9 ms^{-1} (C) 14.7 ms^{-1} (D) 2.8 ms^{-1}

Q.17 A ball is projected from the top of a tower at an angle 60º with the vertical. What happens to the vertical component of its velocity ?
(A) Increases continuously (B) Decreases continuously
(C) Remains unchanged (D) First decreases and then increases.

Q.18 A plane surface is inclined making an angle β above the horizontal, a bullet is fired with the point of projection at the bottom of the inclined plane with a velocity u, then the maximum range is given by :
(A) $\frac{u^2}{g}$ (B) $\frac{u^2}{g(1+\sin\beta)}$ (C) $\frac{u^2}{g(1-\sin\beta)}$ (D) $\frac{u^2}{g(1+\cos\beta)}$

Q.19 Balls A and B are thrown from two points lying on the same horizontal plane separated by a distance of 120 m. Which of the following statement(s) is(are) correct-



(A) The balls can never meet.
(B) The balls can meet if the ball B is thrown 1s later.
(C) The balls can meet if the balls are thrown at the same time.
(D) The two balls meet at a height of 45 m.

Q.20 Two particles are projected simultaneously in the same vertical plane, from the same point, but with different speeds and at different angles with the horizontal. The path followed by one, as seen by the other, is
(A) a vertical straight line
(B) a straight line making a constant angle ($\neq 90^\circ$) with the horizontal
(C) a parabola
(D) a hyperbola

Q.21 Two particles are projected simultaneously in the same vertical plane from the same point, with different speeds u_1 and u_2, making angles θ_1 and θ_2 respectively with the horizontal, such that $u_1\cos\theta_1 = u_2\cos\theta_2$. The path followed by one, as seen by the other (as long as both are in flight), is
(A) a horizontal straight line
(B) a vertical straight line
(C) a parabola
(D) a straight line making an angle $|\theta_1 - \theta_2|$ with the horizontal

Q.22 For a given velocity of projection from a point on the inclined plane, the maximum range down the plane is three times the maximum range up the incline. Then, the angle of inclination of the incline is-
(A) 30º (B) 45º (C) 60º (D) 90º.

ANSWER KEY

LEVEL # 1

Que.	1	2	3	4	5	6	7	8	9
Ans.	A	B	C	D	A	C	A	A	B

LEVEL # 2

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	A	C	D	B	B	A	B	B	A	A	B	A	B
Que.	16	17	18	19	20	21	22								
Ans.	A	D	B	C,D	B	B	A								