Problems Based on Motion Under Gravity

VERY SHORT ANSWER TYPE QUESTIONS :

Q.1 A player throws a ball upwards with an initial speed of $29.4 ms^{-1}$, what is the direction of acceleration during the upward motion of the ball.

Sol. Downward

Q.2 In above question, what are the velocity and acceleration of the ball at the highest points of its motion?

Sol. At the highest point of its motion the velocity is zero and acceleration is $9.8 m/s^{-2}$ acting in downward direction.

Q.3 Is it possible that the velocity of an object be in a direction other than the direction of accelerations.

Sol. Yes. When an object in thrown up, the direction of acceleration due to gravity is downward and direction of velocity is upward.

Q.4 If the instantaneous velocity of the particle is zero, will its instantaneous acceleration be zero?

Sol. No, when an object is thrown up, at the highest point, the instantaneous velocity is zero but instantaneous acceleration is not zero.

Q.5 An iron ball and a wooden ball of same radius are released from a height h in vacuum. Which of the two balls will take less time, to cover a particular distance?

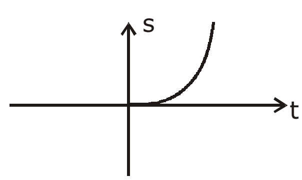
Sol. Both balls will take same time.

SHORT ANSWER TYPE QUESTIONS :

Q.6 Discuss the motion of an object under free fall. Neglect air resistance.

Sol. An object released near the surface of the Earth is accelerated downward under the influence of the force of gravity. The acceleration due to gravity is represented by g. If air resistance is neglected, the object is said to be in free fall. If the height through which the object falls is small compared to the Earth's radius, g can be taken to be constant, equal to $9.8 ms^{-2}$. Free fall is thus a case of motion with uniform acceleration.

Q.7 Figure shows the (S-t) plot of one dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for $t < 0$ and on a parabolic path for $t > 0$? If not suggest a suitable physical context for this graph.



Sol. No, the s-t graph does not show that trajectory of the path of a particle. This graph can be of a particle dropped from a high building or tower. i.e. accelerated motion.

LONG ANSWER TYPE QUESTIONS :

Q.8 Galileo's law of odd numbers. 'The distance traversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as the odd numbers beginning with unity [namely, 1 : 3 : 5 : 7]. Prove it.

Sol. Let us divide the time interval of motion of an object under free fall into many equal intervals τ and find out the distances traversed during successive intervals of time. Since initial velocity is zero,

$$\therefore \quad y = -\frac{1}{2}gt^2$$

Using the equation, we can calculate the positions of the object after different time intervals, o, τ , 2τ, 3τ ... which are given in second column of Table given below. If we take $(-1/2)^2$ $g\tau^2$ as y_0 the position coordinate after first time interval τ, then third column gives the positions in the unit of y_0. The fourth column gives the distances traversed in successive τ's. We fin the distances are in the simple ratio 1 : 3 : 5 : 7 : 9 : 11.. as shown in the last column. This law was established by Galilo Galieli (1564–1642) who was the first to make quantitative studies of free fall.

t	y	y in terms of $y_0[=1/2)g\tau^2]$	Distance traversed in successive intervals	Ratio of distances traversed
0	0	0		
τ	$-(1/2)g\tau^2$	y_0	y_0	1
2τ	$-4(1/2)g\tau^2$	$4y_0$	$3y_0$	3
3τ	$-9(1/2)g\tau^2$	$9y_0$	$5y_0$	5
4τ	$-16(1/2)g\tau^2$	$16y_0$	$7y_0$	7
5τ	$-25(1/2)g\tau^2$	$25y_0$	$9y_0$	9
6τ	$-36(1/2)g\tau^2$	$36y_0$	$11y_0$	11

NUMERICALS :

Q.9 A ball is dropped from a height of 90m on a floor. At each collision with the floor, the ball loses one-tenth of its speed. Plot of speed time graph of its motion between t = 0 to 12s. Take $g = 10ms^{-2}$.

Sol. $u = 0$, $a = 10ms^{-2}$, $S = 90m$, $t = ?$ $v = ?$

Using $v^2 - u^2 = 2aS$, $v^2 - 0^2 = 2 \times 10 \times 90$

or $\quad v = 30\sqrt{2}\ ms^{-1}$

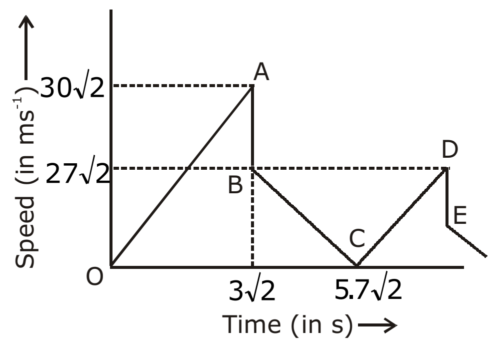
Again, using $S = ut + \frac{1}{2}at^2$, $90 = 0 \times t + \frac{1}{2} \times 10t^2$ or $t = \sqrt{18}\,s = 3\sqrt{2}\,s$

Rebound velocity $= \frac{9}{10} \times 30\sqrt{2}\ ms^{-1}$

$= 27\sqrt{2} ms^{-1}$

Time taken to reach highest point $= \frac{27\sqrt{2}}{10} s = 2.7\sqrt{2}\,s$

Total time $= 3\sqrt{2} + 2.7\sqrt{2} = 5.7\sqrt{2}\,s$



OA represents the vertically downward motion after the ball has been dropped from a height of 90m. The ball reaches the floor with a velocity of $30\sqrt{2}ms^{-1}$ after having been in motion for $3\sqrt{2}\,s$. The vertical straight portion AB represents the loss of $\frac{1}{10}$th of speed. BC represents the vertically upward motion after first rebound. The ball reaches the highest point in $2.7\sqrt{2}\,s$. The total time from the beginning is $3\sqrt{2} + 2.7\sqrt{2}$ i.e., $5.7\sqrt{2}\,s$. C represents the highest points reaches after first rebound. CD represents the vertically downward motion. D represents the situation when the ball again reaches the floor. DE represents the loss of speed.

Q.10 From the top of tower 100m in height a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of $25ms^{-1}$. Find when and where the two balls meet. ($g = 9.8ms^{-1}$)

Sol. Let the two balls meet at a distance h from the top of the tower after times

$h = 0 + \frac{1}{2}gt^2$

or $h = 4.9t^2$

For the body projected vertically upward

$$(100 - h) = 25t - \frac{1}{2} \times 9.8t^2$$

$$(100 - h) = 25t - 4.9t^2$$

or $(100 - h) = 25t - h$ [using equations]

or $t = 4s$

Also $h = \frac{1}{2} \times 9.8 \times 4 \times 4$

$= 78.4m$

Thus the two balls with meet after 4 seconds at a distance of 78.4m from the top of the tower or 21.6m from the bottom of the tower.

Q.11 From the top a building 39.2m, ball is thrown vertically upward with a velocity $9.8ms^{-1}$ find the time when the ball will hit the ground.

Sol. Here $u = +9.8ms^{-1}$

$h = -39.2m$

$g = -9.8ms^{-2}$

Using second equation of motion

$$h = ut + \frac{1}{2}gt^2$$

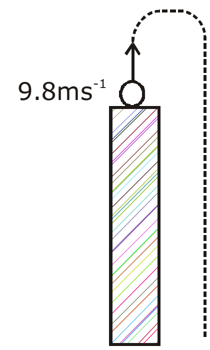
$$-39.2 = 9.8t - \frac{1}{2} \times 9.8t^2$$

or $4.9t^2 - 9.8t - 39.2 = 0$

or $(t - 2)(t - 4) = 0$

or $t = -2$ sec, 4sec

$t = 4$sec



As time cannot be -ve

Q.12 A player throws a ball upward with an initial speed of $29.4ms^{-1}$

(a) What is the direction of acceleration during the upward motion of the ball?

(b) What are the velocity and acceleration of the ball at the highest points of its motion.

(c) To what height does the ball rise and after how along does the ball return to the player's hands? $g = 9.8ms^{-2}$

Sol. (a) The acceleration is due to gravity which acts downward

(b) At the highest point of its motion

velocity = 0

acceleration = $9.8ms^{-2}$

(c) Here $u = 29.4ms^{-1}$

$v = 0$

$g = -9.8ms^{-2}$

From 3rd equation of motion

$$v^2 = u^2 + 2gh$$

$$0 = (29.4)^2 - 2 \times 9.8h$$

or $h = \frac{(29.4)^2}{2 \times 9.8} = 44.1m$

Problems Based on Motion Under Gravity

Q.13 A ball thrown vertically upward with a velocity of $19.6ms^{-2}$ from the top of a tower returns to the ground in 6s. Find the height of the tower.

Sol. Here $u = -19.6ms^{-1}$

(In upward direction velocity is taken -ve)

$t = 6s$

From 2nd equation of motion

$$h = ut + \frac{1}{2}gt^2$$

$$h = -19.6 \times 6 + \frac{1}{2} \times 9.8 \times (6 \times 6)$$

$= -117.6 + 176.4$

$= 58.8m$

Q.14 A balloon is ascending at the rate of $14ms^{-1}$. At a height of 98m above the ground a packet is dropped from the balloon. After how long and with what velocity does it reach the ground?

Sol. Here $u = -14ms^{-1}$

$h = 98m$

$g = 9.8ms^{-2}$

$t = ?$ $v = ?$

(i) From 3rd equation of motion

$v^2 = u^2 + 2gh$

$v^2 = (-14)^2 + 2(9.8)(98)$

$v^2 = 196 + 1920$

or $v = 46ms^{-1}$

(ii) Also $v = u + gt$

or $t = \frac{v - u}{9.8}$

$$\frac{46 - (-14)}{9.8} = 6.12s$$

Q.15 Four balls are dropped gently from the top of a tower at intervals of one-one second. The first ball reaches the ground after 4 seconds of dropping. What are the distances between first and second, and third, and fourth ball at this instant? ($g = 9.8ms^{-2}$)

Sol. From second equation of motion

$$h = ut + \frac{1}{2}gt^2$$

For 1st ball $h_1 = 0 + \frac{1}{2} \times 9.8 (4)^2 = 78.4m$

For 2nt ball $h_2 = 0 + \frac{1}{2} \times 9.8 (3)^2 = 44.1m$

For 3rd ball $h_3 = 0 + \frac{1}{2} \times 9.8 (2)^2 = 19.6m$

For 4st ball $h_4 = 0 + \frac{1}{2} \times 9.8\ (1)^2 = 4.9m$

Distance between 1st and 2nd ball is

$h_1 - h_2 = 78.4 - 44.1 = 34.3m$

Distance between 2st and 3rd ball is

$h_2 - h_3 = 44.1 - 19.6 = 24.5m$

Distance between 3st and 4nd ball is

$h_3 - h_4 = 19.6 - 4.9 = 14.39m$

Q.16 Two balls are thrown simultaneously. A vertically upwards with a speed of $20ms^{-1}$ from the ground and B vertically downwards from a height of 40m with the same speed and along the same line of motion. At what points do the two balls collide? ($g = 9.8ms^{-2}$)

Sol. For ball A : $u = 20ms^{-1}$

$g = 9.8ms^{-2}$

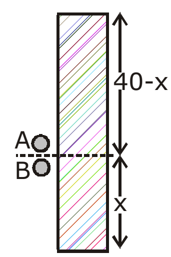
For ball B $u = 20ms^{-1}$

$g = -9.8ms^{-2}$

Consider, two balls collide at a distance x from ground after time t.

From second equation of motion

$$h = ut + \frac{1}{2}gt^2$$



For ball A $(40 - x) = 20t + \frac{1}{2} \times 9.8^2$ .....(1)

For ball B $x = 20t + \frac{1}{2}(-9.8)t^2$ .....(2)

Adding (1) and (2) $40 = 40t$

or $t = 1sec.$

Also $x = 20 \times 1 + \frac{1}{2}(-9.8)\ (1)^2$

$= 15.1m$

Q.17 A body dropped from the top of a tower falls through 40m during the last two seconds of its fall. What is the height of the tower ? ($g = 10ms^{-2}$)

Sol. Let h be the height of the tower and t is the time of ball.

Here $u = 0$, $g = 10ms^{-2}$

From second equation of motion

$$h = ut + \frac{1}{2}gt^2$$

$$h = 0 + \frac{1}{2} \times 10^2$$

$$h = 5t^2 \quad .....(1)$$

The body travels 40m during the last two seconds. Therefore the body covers (h – 40) metre in (t – 2) seconds.

$\therefore \quad (h - 40) = 0 + \frac{1}{2} \times 10\,(t-2)^2$

$(h - 40) = 5\,(t-2)^2 \quad .....(2)$

Using equation (1)

$5t^2 - 40 = 5\,(t-2)^2$

or $\quad [t^2 - (t-2)^2] = 8$

or $\quad 4t - 4 = 8$

$4t = 12$

$t = 3s.$

$\therefore \quad h = 5t^2 = 5 \times 3 \times 3 = 45m$

Q.18 A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to $49ms^{-1}$. (a) How much time does the ball take to returns to his hands ? (b) If the left starts moving up with a uniform speed of $5ms^{-1}$, and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands?

Sol. (a) $v(0) = 49ms^{-1}$, $a = -9.8ms^{-2}$, $t = ?$, $v(t) = 0$

$v(t) = v(0) + at$

$0 = 49 - 9.8t$ or $9.8t = 49$ or $t = \frac{49}{9.8}s = 5s$

This is the time taken by the ball to reach the maximum height. The time of descent is also 5s. So, the total time after which the ball comes back is 5s + 5s i.e., 10s

(b) The uniform velocity of the lift does not change the relative motion of ball and lift. So, the ball would take the same total time i.e., it would come back after 10second.

Alter. 'u' = $49ms^{-1}$, 'a' = $-9.8ms^{-2}$, t = ?, v = 0

Using v = u at, $0 = 49 - 9.8t$ or $9.8t = 49$ or $t = \frac{49}{9.8}s = 5s$

Q.19 A ball if dropped from a height of 5metre on a plane. On bouncing , it rises to a height of 1.8 metre. Calculate the fractional loss of velocity of ball. The value of g in not known.

Sol. Let v_1 be the velocity of reaching the ground $v_1 = \sqrt{2g \times 5}\,ms^{-1}$

After bouncing, the velocity of the body is v_2. $v_2 = \sqrt{2g \times 1.8}\,ms^{-1}$

Dividing, $\quad \frac{v_2}{v_1} = \sqrt{\frac{1.8}{5}} = \sqrt{\frac{18}{50}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$

Fractional loss of velocity = $\frac{v_1 - v_2}{v_1} = 1 - \frac{v_2}{v_1} = 1 - \frac{3}{5} = \frac{2}{5} = 0.4$

Problems Based on Motion Under Gravity

Q.20 You can measure your reaction time by a simple experiment. Take a ruler and ask your friend to drop it vertically through the gap between your thumb and forefinger. As soon as it is dropped, note the time elapsed before you catch it and the distance travelled by the ruler. In a particular case, the distance was found to be 21.0cm. Estimate the reaction time.

Sol. The rule drops under free fall.

$\therefore$ $u = 0,\ a = g = 9.8 ms^{-2}$

If d is the distance travelled and t_r is the reaction time, then using

$$S = ut + \frac{1}{2}at^2,$$

We get $d = 0 \times t_r + \frac{1}{2} g_0 t_r^2$

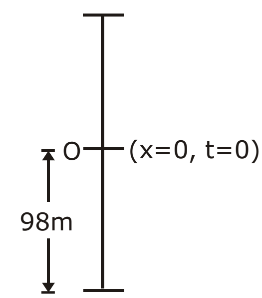
or $t_r = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2 \times 0.21}{9.8}} s = 0.207s$

Q.21 A balloon is ascending at the rate of $14ms^{-1}$ at a height of 98m above the ground when a packet is dropped from the balloon. After how much time and with what velocity does it reach the ground?

Sol. Let O be the origin and the vertically upward direction be the positive direction of x-axis.

At O, $x = 0,\ t = 0$

Height of O above the ground in 98m.



When the packet is dropped from the balloon, it has the same velocity as that of the balloon, i.e., $14ms^{-1}$ in the vertically upward direction.

$\therefore$ $v(0) = 14ms^{-1}$

Acceleration due to gravity will act in the vertically downward direction

$\therefore$ $a = -g = -9.8ms^2$

In order to reach the ground, the packet has to cover a vertically downward distance of 98m. If the packet does so in time t, then

$x(t) = -98m$

Now, $x(t) = x(0) + v(0)\,t + \frac{1}{2}at^2$

or $-98 = 0 + 14t + \frac{1}{2}(-9.8)t^2$

or $-98 = 14t - 4.9t^2$

or $4.9t^2 - 14t - 98 = 0$

or $49t^2 - 140t - 980 = 0$

or $7t^2 - 20t - 140 = 0$

$\therefore$ $t = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(7)(-140)}}{14} s = \frac{20 \pm \sqrt{400 + 3920}}{14} s = \frac{20 \pm \sqrt{4320}}{14} = \frac{20 \pm 65.727}{14} s$

Ignoring –ve value, $t = \frac{20 + 65.727}{14} s = 6.12s$

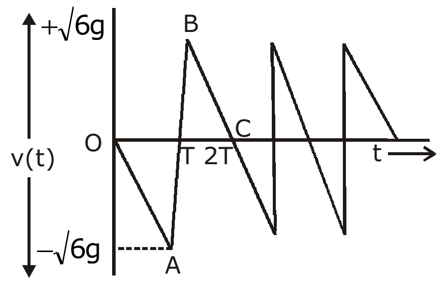
Let $v(t) = v(0) + at = (14 - 9.8 \times 6.12)ms^{-1} = -46ms^{-1}$

Negative sign indicates a vertically downward velocity.

Q.22 A metal ball is allowed to fall freely on a perfectly elastic plate from a height of 3m. At t = 0, the speed of the ball is zero. Represent graphically the variation of velocity with time.

Sol. Suppose the ball takes time T to reach the plate.

Let the origin be chosen at a height of 3m above the ground. Let us consider velocities directed vertically upwards as positive and those directed downwards as negative.



$x(t) = -3m$, $x(0) = 0$, $v(0) = 0$, 't' = T, 'a' = –g

Now, $x(t) = x(0) + v(0)t + \frac{1}{2}at^2$

$\therefore \quad -3 = 0 + 0 - \frac{1}{2}gT^2$

$T^2 = 6/g$ or $T = \sqrt{\frac{6}{g}}$

The velocity of the ball at any time t, while moving downwards, is

$v(t) = 0 - gt \qquad [\because v(t) = v(0) + at]$

AT t = T, $v(T) = -gT = -g\sqrt{\frac{6}{g}} = -\sqrt{6g}$

The part OA of the graph represents the 'first bounces' of the ball. The slope of this graph is '–g'. The point A corresponds to velocity $-\sqrt{6g}$

When the ball collides against the plate, its velocity is reversed, i.e., its velocity changes from $-\sqrt{6g}$ to $\sqrt{6g}$.

from t = T to t = 2T, the velocity continues the positive. At t = 2T, the ball reaches its initial height to 3m and its velocity becomes zero. This is represented by the portion BC of the graph.

As the ball continues to fall and rise, the part OABC of the graph continues to repeat itself.