Problems Based on Uniform Circular Motion

VERY SHORT ANSWER TYPE QUESTIONS :

Q.1 Give an example of motion in which displacement and distances are not equal but speed remains constant during the motion.

Sol. Uniform circular motion.

Q.2 Can an object be accelerated without speeding up or slowing down?

Sol. Yes. When an object is in uniform circular motion, its speed remains constant but there is an acceleration towards centre of circular path.

Q.3 Write S.I. unit and dimensions of angular velocity.

Sol. Radian per second (rad s^{-1}) Dimensions $[T^{-1}]$

Q.4 What do you mean by uniform angular velocity.

Sol. If a body covers equal angles intervals of time then the angular velocity of the body is called uniform angular velocity.

Q.5 Define average angular acceleration.

Sol. Average angular acceleration is defined as the ratio of the change in angular velocity to the time taken by the body to undergo this change i.e.

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

Q.6 Write the S.I. unit and dimensions of angular acceleration.

Sol. Unit radian sec^{-2}. Dimensions $[M^0L^0T^{-2}]$

Q.7 What is the direction of velocity vector of a particle in uniform circular motion.

Sol. The direction of velocity vector of a particle in uniform circular motion is along the tangent to the circular path.

Q.8 Uniform circular motion is an example of accelerated motion. Give reason.

Sol. Yes. uniform circular motion is accelerated motion. In this motion the direction of velocity changes continuously.

Q.9 What provides the centripetal force to satellite revolving around the earth?

Sol. The centripetal force of the satellite revolving around the earth is provided by the gravitational force between the earth and satellite.

Q.10 Write the relation between, the linear velocity (v), angular velocity (ω) and radius of the circular path (r).

Sol. $v = r\omega$

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Q.11 Can a centripetal force do any work on the rotating body?

Sol. No, as the centripetal force is always perpendicular to the displacement.

Q.12 Is it possible to accelerate a body if its speed is constant?

Sol. Yes, in circular motion the speed of a body is constant but acceleration is not zero.

Q.13 Write an expression for centripetal force

Sol. $a = \frac{v^2}{r} = r\omega^2$

Q.14 What do you mean by centrifugal force?

Sol. The outward radial force experienced by an object when it is in circular motion is called centrifugal force.

Q.15 A body subjected to uniform acceleration always moves in a straight line. Is the statement true?

Sol. No.

Q.16 Why do the passengers of a car rounding a curve are thrown outward?

Sol. Due to centrifugal force

Q.17 What are the two physical quantities which remain constant for a particle performing uniform circular motion.

Sol. Speed and magnitude of centripetal acceleration.

SHORT ANSWER TYPE QUESTIONS :

Q.18 If the speed of a body and the radius of its circular path are made thrice by what factor the centripetal acceleration will change?

Sol. $a = \frac{v^2}{r}$

when the speed and the radius are made thrice then

$$a' = \frac{(3v)^2}{3r} = \frac{3v^2}{r} = 3a$$

Q.19 Give two examples of uniform circular motion.

Sol. (a) The motion of the minute hand of a watch.

(b) The motion of the blades of the fan when the electric fan has attained constant speed.

Q.20 Can Earth be regarded as a point object if only the orbital motion of Earth around the Sun in considered?

Sol. Yes. This is because the size of the Earth is very small as compared to the size of the orbit of the Earth around the Sun.

Q.21 Define average angular velocity.

Sol. Average angular velocity is defined as the ratio of the angular displacement to the time taken to undergo that angular displacement.

Q.22 A body is moving with an acceleration whose magnitude is constant but direction is changing continuously. What type of path does the body follow?

Sol. The body follows a circular path.

LONG ANSWER TYPE QUESTIONS :

Q.23 Define centripetal acceleration and derive an expression for it.

Sol. Acceleration experienced by an object, undergoing uniform circular motion is called centripetal acceleration. This acceleration always points towards the centre of circular path.

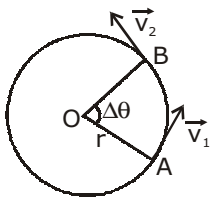
Consider a particle of mass m, moving with a constant speed v along a circular path of radius r. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocity vectors of the particle at positions A and B respectively. Since the particle is moving with uniform speed v.

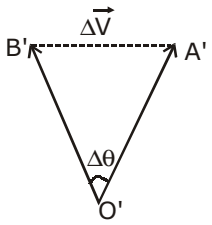
$\therefore \quad |\vec{v}_1| = |\vec{v}_2| = v$

To find the change in velocity in time interval Δt take a point O'. Draw $\overrightarrow{O'A'}$ and $\overrightarrow{O'B'}$ representing the velocity vectors $\vec{v}_1$ and $\vec{v}_2$. $\angle A'O'B' = \theta$

$\overrightarrow{A'B'}$ represents the change in velocity $\Delta\vec{V}$

As $\Delta\theta$ is small





$$\Delta\theta = \left|\frac{\Delta\vec{V}}{v^2}\right|$$

or $$\Delta\theta = \left|\frac{\Delta\vec{v}}{\Delta t}\right| = \omega\,|\vec{v}| = (r\omega) = r\omega^2$$

$$\text{Limit}_{\Delta t \to 0} \left|\frac{\Delta\vec{v}}{\Delta t}\right| = r\omega^2$$

or $$\left|\frac{d\vec{v}}{dt}\right| = r\omega^2$$

or $$|\vec{a}| = r\omega^2 = \frac{V^2}{r}$$

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NUMERICALS :

Q.24 A passenger arriving in a new town wishes to go from the station to a hotel located 10km away on a straight road from the station. A dishonest cab man takes him along a circuitous path 23km long and reaches the hotel in 28min. What is (a) the average speed of the taxi. (b) The magnitude of average velocity? Are the two equal?

Sol. Here Total path covered = 23km

Displacement = 10km

Time taken = 28min

$$\text{Average speed} = \frac{\text{Total path covered}}{\text{Time taken}}$$

$$= \frac{23}{28/60}$$

$$= \frac{23 \times 60}{28}$$

$$= 49.3 \text{kmh}^{-1}$$

Magnitude of average velocity

$$= \frac{\text{Displacement}}{\text{Time taken}}$$

$$= \frac{10}{28/60}$$

$$= \frac{10 \times 60}{28} = 21.42 \text{kmh}^{-1}$$

Q.25 A stone tied to the end of a string 80cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25seconds what is the magnitude and direction of the acceleration of the stone?

Sol. Here $r = 0.80\text{m}$

$$v = \frac{14}{25}\text{rps.}$$

Form formula $a = r\omega^2 = 4\pi^2 v^2 r$

$$= 4 \times \left(\frac{22}{7}\right)^2 \times \left(\frac{14}{25}\right)^2 (0.80)$$

$$= 9.91 \text{ms}^{-2}$$

The acceleration is pointing towards the centre of circle.

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Q.26 An aircraft executes a horizontal loop of radius 1km with a steady speed of 900kmh^{-1}, compare the centripetal acceleration with the acceleration due to gravity.

Sol. Here $r = 1\ km = 1000m$

$v = 900ms^{-1}$

$= 900 \times \frac{5}{18} = 250ms^{-1}$

Centripetal acceleration $= \frac{v^2}{r} = \frac{(250)^2}{1000} = 62.50ms^{-2}$

Now $\frac{a}{g} = \frac{62.50}{9.8} = 6.38$

or $a = 6.38g$

Q.27 An insect trapped in a circular groove of radius 12cm moves along the groove steadily and complete 7 revolutions in 100s. (a) What is the angular speed and the linear speed of the motion? (a) Is the acceleration vector a constant vector? What is the magnitude?

Sol. The given problem is related to uniform circular motion.

Radius, R = 12cm, angular speed, $\omega = \frac{2\pi}{T}$

or $\omega = 2 \times \frac{22}{7} \times \frac{7}{100} = 0.44 rad s^{-1}$

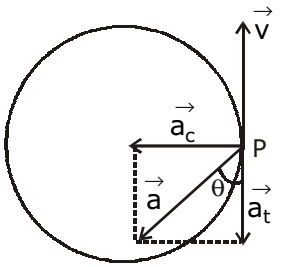
Linear speed, $v = R\omega = 12 \times 0.44cms^{-1} = 5.3cms^{-1}$

The direction of velocity vector $\vec{v}$ is along the tangent to the circle at every point. The acceleration is directed towards the center of the circle. Since the direction changes continuously therefore acceleration is not a constant vector. But the magnitude of acceleration is a constant.

$\therefore$ $a = \omega^2 R = (0.44)^2 \times 12cms^{-2} = 2.3cms^{-2}$

Q.28 A cyclist is riding with a speed of 27kmh^{-1}. As he approaches a circular turn on the road of radius 80m, every on the road of radius 80m. What is the magnitude and direction of the net acceleration of the cyclist on the circular turn?

Sol. Speed, $v = 27kmh^{-1} = 27 \times \frac{5}{18} ms^{-1} = 7.5ms^{-1}$



centripetal acceleration, $a_c = \frac{v^2}{r}$

or $a_c = \frac{(7.5)^2}{80} ms^{-2} = 0.7ms^{-2}$

P is the point at which cyclist applies brakes. At this point, tangential acceleration a_t, being negative, will act opposite to $\vec{v}$

Total acceleration, $a = \sqrt{a_c^2 + a_t^2}$

or $a = \sqrt{(0.7)^2 + (0.5)^2}\,ms^{-2} = \sqrt{0.49 + 0.25}\,ms^{-2}$

$= \sqrt{0.74}\,ms^{-2} = 0.86 ms^{-2}$

$\tan\theta = \dfrac{a_c}{a_t} = \dfrac{0.7}{0.5} = 1.4$

$\therefore$ $\theta = 54^o28'$