Problems Based on Relative Motion

VERY SHORT ANSWER TYPE QUESTIONS :

Q.1 Can a body at rest be regarded in a state of motion? Give example.

Sol. Yes. A cup placed on the table is at rest with respect to table but is in motion with respect to a girl moving on the floor.

Q.2 Can a body be said to be are rest as well as in motion at the same time.

Sol. Yes. A passenger in a train is at rest to other passenger but is in motion with respect to the surroundings.

Q.3 Is it true that a body is always at rest in a frame which is fixed to the body itself.

Sol. Yes.

SHORT ANSWER TYPE - I QUESTIONS :

Q.4 Two objects A and B start moving towards each other with velocities $20ms^{-1}$ and $25ms^{-1}$ respectively from the separation of 600m. What is the displacement of A w.r.t B after 4sec.

Sol. $S_A = 20 \times 4 = 80m$

$S_B = -25 \times 4 = -100m$

Displacement of A w.r.t. B = $S_A - S_B$

$= 80 - (-100) = 180m$

Q.5 Define relative velocity and write its S.I. unit.

Sol. The relative velocity of an object A with respect another object B (both the objects may be in motion) is defined as the rate of change of position of object A with respect to B.

Let velocity of object A is $\vec{V}_A$ and the velocity object B is $\vec{V}_B$, then the relative velocity of A with respect to B is

$$V_{AB} = \vec{V}_A - \vec{V}_B$$

Q.6 An automobile travelling with a speed of $50kmhr^{-1}$ on a straight road ahead of a scooter traveling with a speed of $60kmhr^{-1}$. How would the relative velocity be altered if scoter is ahead of automobile?

Sol. The relative velocity does not depend upon the position of objects. There relative velocity will remain same.

Q.7 Two bodies are moving in opposite direction. The relative velocity will be the sum of their individual velocities. Does it necessary that their separation is decreasing?

Sol. It is not necessary that the separation of two bodies is decreasing when they are moving in opposite directions and the relative velocity is the sum of their individual velocities

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In following diagram the bodies are moving in opposite directions but their separation is increasing.

The relative velocity is still the sum of their individual velocities.

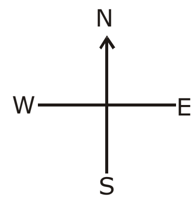
Q.8 A car travelling with a sped of $60kmh^{-1}$ on a straight road is ahead of a scooter travelling at a speed of $40kmh^{-1}$. How would the relative velocity be altered if scooter is ahead of the car?

Sol. The magnitude of relative velocity in both the cases is $20kmh^{-1}$. However the direction will be reversed.

LONG ANSWER TYPE QUESTIONS :

Q.9 A bird flying at velocity $4ms^{-1}$ in east direction. The wind is blowing towards north at a velocity of $3ms^{-1}$. Calculate relative velocity of bird with respect to wind.

Sol. Relative velocity of bird w.r.t. wind



$V = V_b - V_w$

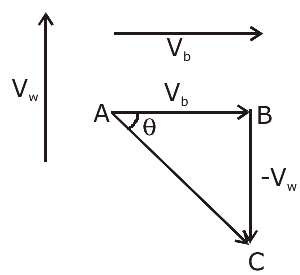
$= V_b + (-V_w)$

The magnitude of V

$V = \sqrt{V_b^2 + V_w^2}$

$= \sqrt{(3)^2 + (4)^2}$]

$= 5ms^{-1}$



Also $\tan\theta = \dfrac{3}{4}$

or $\theta = \tan^{-1}(3/4)$

(There vector diagram for the problem is shown in figure). $\vec{AC}$ shows resultant velocity (v)

NUMERICALS :

Q.10 How long will a boy sitting near the window of a train travelling at $54kmh^{-1}$ see a train passing by in the opposite direction with a speed of $36kmh^{-1}$. The length of the slow-moving train is 100m.

Sol. Relative velocity of the slow moving train w.r.t the boy

$= (54 + 36)\ kmh^{-1} = 90kmh^{-1}$

$= 90 \times \dfrac{5}{18} msh^{-1} = 25ms^{-1}$ $\quad 1kmh^{-1} \dfrac{5}{18} ms^{-1}$

Now, $25 = \dfrac{100}{t}$ or $t = \dfrac{100}{25} s = 4s$

Problems Based on Relative Motion

Q.11 A swimmer's speed in the direction of flow of river is $16kmh^{-1}$. Against the direction of flow of river, the swimmer's speed is $8kmh^{-1}$. Calculate the swimmer's speed in still water and the velocity of flow of the river.

Sol. Let v_s and v_r represent the velocities of swimmer and river respectively.

Now, $v_s + v_r = 16$

and $v_s - v_r = 8$ .....(1)

Adding, $2v_s = 16 + 8 = 24kmh^{-1}$

or $v_s = 12kmh^{-1}$

From Eq. (1) $12 + v_r = 16$

or $v_r = 4kmh^{-1}$

Q.12 A train 110m long is traveling at $60kmh^{-1}$. In what time it will cross a cyclist moving at $6kmh^{-1}$ (a) in the same direction, (b) in the opposite direction?

Sol. Velocity of train, $v_t = 60kmh^{-1}$;

Velocity of cyclist, $v_c = 6kmh^{-1}$

(a) Relative velocity of train w.r.t cyclist, $v_{tc} = (60 - 6)kmh^{-1}$

$$= 54kmh^{-1} = 54 \times \frac{5}{18}ms^{-1} = 15ms^{-1}$$

Now, $15 = \frac{100}{t}$ or $t = \frac{100}{15}s = 7.33s$

(b) Relative velocity of train w.r.t cyclist, $v_{tc} = (60 + 6)kmh^{-1} = 66kmh^{-1} = 66 \times \frac{5}{18}$

Now, $t = \frac{110 \times 18}{60 \times 5}s = 6s$

Q.13 A police van moving on a highway with a speed of $30kmh^{-1}$ fires a bullet at a thief's car speeding away in the same direction with a speed of $192kmh^{-1}$. If the muzzle speed of the bullet is $150kmh^{-1}$, with what speed does the bullet hit the thief's car?

Sol. Speed of police van,

$$v_p = 30kmh^{-1} = \frac{30 \times 1000}{3600} ms^{-1} = \frac{25}{3}ms^{-1}$$

Speed of thief's car,

$$v_t = 192kmh^{-1} = \frac{192 \times 1000}{3600} ms^{-1} = \frac{160}{3}ms^{-1}$$

Speed of bullet, v_b = speed of police van + speed with which bullet is actually fired

$$\therefore \quad v_b = \left(\frac{25}{3} + 150\right)ms^{-1} = \frac{475}{3}ms^{-1}$$

Relative velocity of bullet w.r.t. thief's car,

$$v_{bt} = v_b - v_t = \left(\frac{475}{3} - \frac{160}{3}\right)ms^{-1} = 105ms^{-1}$$

Problems Based on Relative Motion

Q.14 A jet air plane traveling at the speed of 500kmh^{-1} ejects its product of combustion at the speed of 1500kmh^{-1} relative to the jet plane. What is the speed of the combustion products w.r.t an observer on the ground?

Sol. Speed of combustion products w.r.t observer on the ground = ?

Velocity of jet air plane w.r.t observer on ground = 500kmh^{-1}

If $\vec{v}_j$ and $\vec{v}_0$ represent the velocities of jet and observer respectively, then $v_j - v_0 = 500\text{kmh}^{-1}$

Similarly, if $\vec{v}_c$ represents the velocity of the combustion product w.r.t. jet plane, then

$v_c - v_j = -1500\text{kmh}^{-1}$

The negative sign indicates that the combustion products move in a direction opposite to that of jet.

Speed of combustion products w.r.t. observer

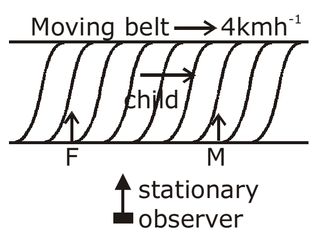
$= v_c - v_0 = (v_c - v_j) + (v_j - v_0) = (-1500 + 500)\text{kmh}^{-1} = -1000\text{kmh}^{-1}$

Q.15 On a long horizontal moving belt, a child runs to and fro with a speed 9kmh^{-1} (with respect to the belt) between this father and mother located 50m apart on the moving belt. The belt moves with a speed of 4kmh^{-1}. For an observer on a station platform outside, what is the

(a) speed of the child running in the direction of motion of the belt?

(b) speed of the child running opposite to the direction of motion of the belt?

(c) time taken by the child in (a) and (b)? Which of the answers alter if motion is viewed by one of the parent?



Sol. Speed of child with respect to belt = 9kmh^{-1} speed of belt = 4kmh^{-1}

(a) When a child runs in the direction of motion of the belt, then speed of child w.r.t. stationary observer = (9 + 4) kmh^{-1} = 13kmh^{-1}

(b) When the child runs opposite to the direction of motion of the belt, then speed of child w.r.t. stationary observer = (9 −4) kmh^{-1} = 5kmh^{-1}

(c) speed of child w.r.t either parent = 9kmh^{-1}

Distance to be covered = 50m = 0.05km;

Time $= \dfrac{0.05\text{km}}{9\text{kmh}^{-1}} = 0.0056\text{h} = 20\text{s}$

If the motion is viewed by one of the parents, then the answers to (a) and (b) are altered but answer to (c) remains unaltered.

Problems Based on Relative Motion

Q.16 The speed of a motor launch with respect to still water is $7ms^{-1}$ and the speed of the stream is $3ms^{-1}$. When the launch began travelling upstream, a float was dropped from it. The launch travelled 4.2 km upstream, turned about and caught up with the float. How long is it before the launch reached the float?

Sol. Relative velocity of the launch while travelling upstream = launch velocity - stream velocity $= (7 - 3)\ ms^{-1} = 4ms^{-1}$

Time taken by the launch for travelling a distance of 4.2km, $t_1 = \dfrac{4.2\times10^3 m}{4ms^{-1}} = 1.05 \times 10^3 s$

Suppose t is the time taken by the launch after dropping the float and meeting it again.

Distance travelled by the float during time $t = 3ms^{-1} \times ts = 3t$ meter

Relative velocity of the launch while travelling downstream $= (7 + 3)ms^{-1} = 10ms^{-1}$

Distance travelled down the stream $= (4.2 \times 10^3 + 3t)$ merte

If t_2 is the time taken to cover this distance, then $t_2 = \dfrac{4.2\times10^3+3t}{10} s$

Again, $t = t_1 + t_2 = 1.05 \times 10^3 + \dfrac{4.2\times10^3+3t}{10}$

On simplification, $t = 2100s = 35$ minute

Q.17 A police van moving on a highway with a speed of $30kmh^{-1}$ fires a bullet at a thief's car speeding away in the same direction with a speed of $19kmh^{-1}$. If the muzzle speed of the bullet is $150ms^{-1}$ with what speed does the bullet hit the thief's car?

Sol. The speed of bullet $= 150ms^{-1}$

$$= \frac{150\times60\times60}{1000} kmh^{-1}$$

$= 540kmh^{-1}$

The speed of police car $= 30kmh^{-1}$

$\therefore$ Effective speed of bullet $= 540 + 20$

$= 570kmh^{-1}$

The speed of theif's car $= 192kmh^{-1}$

$\therefore$ Relative speed of bullet w.r.t. thief's car $= 570 - 192$

$= 378\ kmh^{-1}$

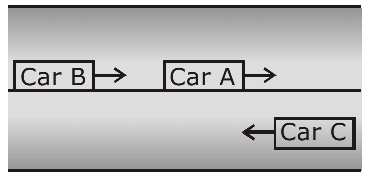
$= 378 \times 5/18\ ms^{-1}$

$= 105ms^{-1}$

Problems Based on Relative Motion

Q.18 On a two lane road, car A is travelling with a speed of $36kmh^{-1}$. Two cars B and C approach car A in opposite directions with a speed of $54kmh^{-1}$. At a certain instant, when the distance AB is equal to AC both being 1km. B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?

Sol. The speed of car A = $36kmh^{-1} = 36 \times \frac{5}{18} = 10ms^{-1}$



The speed of the car B and C = $54kmh^{-1} = 54 \times \frac{5}{18} = 15ms^{-1}$

The relative speed of car B w.r.t car A = 15 – 10

= $5ms^{-1}$

The relative speed of car C w.r.t car A = 15 – (–10)

= $25ms^{-1}$

Time taken by car C to reach to car A = $\frac{\text{Distance}}{\text{Speed}} = \frac{1000}{25} = 40\text{sec.}$

In order to avoid accident car B must over take car A in time less than 40sec.

Using second equation of motion

$$S = ut + \frac{1}{2}at^2$$

$$1000 = 5 \times 40 + \frac{1}{2}a \times 40 \times 40$$

$$\therefore \quad a = \frac{800}{800} = 1ms^{-2}$$

Q.19 Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T min. A man cycling with a speed of $20kmh^{-1}$ in the direction A to B notices that a bus goes past him every 18min in the direction of this motion and every 6min in opposite direction. What is the period T of the bus service and with what speed do the buses ply on the road ?

Sol. Let v be the constant speed with which the buses are plying between the towns A and B.

The cyclist is going from A to B at a speed of $20kmh^{-1}$

The relative speed of the bus going from A to B w.r.t the cyclist = $V - 20kmh^{-1}$

In the time T the bus will cover a distance vT in direction A to B

$vT = 18(v - 20)$

In direction B to A

$vT = 6(v + 20)$

On solving equation (1) and (2)

$V = 40kmh^{-1}$ and $t = 9min.$

Q.20 A jet air plane moving a the speed of $500kmh^{-1}$ ejects its products of combustion at the speed of $15000kmh^{-1}$ relative to it. Find the speed of the latter with respect to an observer on the ground.

Sol. $\therefore$ Velocity of jet plane $V_j = -500kmh^{-1}$

Velocity of product of combustion,

with respect to jet $= V_p = 1500kmh^{-1}$

Relative velocity of product of combustion w.r.t jet

= velocity of product – velocity of jet

$\therefore \quad 1500 = V - (-500)$

or $\quad V = 1500 - 500$

$= 1000kmhr^{-1}$